Appendix 5.A11: Derivation of solar gain factors

5.A11.1 Notation

Symbols used in this appendix are as follows.

a	Fraction of incident energy absorbed by thickness L (mm) of glass
A	Absorption coefficient
A'	Absorption coefficient for double glazing
A''	Absorption coefficient for triple glazing
A_D	Absorption coefficient for direct radiation
A_d	Absorption coefficient for diffuse radiation
A_{dg}	Absorption coefficient for ground reflected radiation
A_{ds}	Absorption coefficient for sky diffuse radiation
C_1, C_2	Configuration factors for slatted blinds
D	Slat thickness (mm)
F	Surface factor
h	Solar altitude (degree)
H	Transmittance factor
I	Incident solar irradiance ($W \cdot m^{-2}$)
j	Number of surface
k	Glass extinction coefficient
L	Glass thickness (mm)
M	Width of slat illuminated (mm)
n	Total number of surfaces
R	Reflection coefficient
R'	Reflection coefficient for double glazing
R''	Reflection coefficient for triple glazing
r	Ratio of incident beam to reflected beam at air/glass interface
$r_{//}$	Ratio of incident beam to reflected beam at air/glass interface for radiation polarised parallel to the plane of incidence
$r_\perp$	Ratio of incident beam to reflected beam at air/glass interface for radiation polarised perpendicular to the plane of incidence
R_D	Reflection coefficient for direct radiation
R_d	Reflection coefficient for diffuse radiation
R_{dg}	Reflection coefficient for ground reflected radiation
R_{ds}	Reflection coefficient for sky diffuse radiation
R_{se}	External surface resistance ($W \cdot m^{-2} \cdot K^{-1}$)
R_{si}	Internal surface resistance ($W \cdot m^{-2} \cdot K^{-1}$)
T	Transmission coefficient
T'	Transmission coefficient for double glazing
T''	Transmission coefficient for triple glazing
T_D	Transmission coefficient for direct radiation
T_d	Transmission coefficient for diffuse radiation
T_{dg}	Transmission coefficient for ground reflected radiation
T_{ds}	Transmission coefficient for sky diffuse radiation
T_n	Transmission coefficient at normal incidence
t	Time (h)
W	Slat width (m)
α	Absorptivity (thermal shortwave radiation)
β	Profile angle (degree)
γ	Wall azimuth (degree)
γ_s	Wall–solar azimuth (degree)
ζ_i	Angle of incidence (degree)
ζ_r	Angle of refraction (degree)
θ	Temperature (°C)
μ	Refractive index of glass (= 1.52)
σ_v	Vertical shadow angle (degree)
Φ	Room gain ($W \cdot m^{-2}$)
Φ_a	Room gain to air node ($W \cdot m^{-2}$)
Φ_{at}	Room gain to air node at time t ($W \cdot m^{-2}$)
Φ_e	Room gain to environmental node ($W \cdot m^{-2}$)
Φ_{et}	Room gain to environmental node at time t ($W \cdot m^{-2}$)
Φ_t	Room gain at time t ($W \cdot m^{-2}$)
ϕ	Solar azimuth (degree)
ψ	Slat angle (degree)
ω	Time lag associated with surface factor (h)

Where required additional subscripts 'A', 'R' and 'T' indicate gains due to absorbed, reflected and transmitted components of radiation, respectively.

5.A11.2 Response of room to solar radiation

Shortwave solar radiation incident upon a window will be reflected, absorbed in the glazing elements or directly transmitted into the space beyond the window. The absorbed radiation will increase the temperature of the glazing and is therefore both a longwave radiant heat gain and a convective gain to the space. In terms of the simple (dynamic) model these gains are considered to enter the model at the environmental node. If internal blinds are present, there will be an increase in the convective portion of the gain which enters the model at the air node. Transmitted radiation must be absorbed at the room surfaces before it can become a heat gain to the space. With the exception of any shortwave radiation that passes directly out of the space by transmission through glazed surfaces, all the radiation entering the space is absorbed at the room surfaces or within the furnishings.

Once absorbed, the radiation warms the surfaces and, after a time delay, enters the space at the environmental node by means of convection and radiation.

For the purposes of the simple (dynamic) model, the room gain is divided into a 24-hour mean component and an hourly cyclic component.

For any given source:

$$\bar{\Phi} = (1/24) \sum_{t=1}^{t=24} \Phi_t \qquad (A11.1)$$

and:

$$\widetilde{\Phi}_t = \Phi_t - \bar{\Phi} \qquad (A11.2)$$

In the case of the simple (dynamic) model the gain will be either to the environmental node only or to both the environmental and air nodes. The gain to the environmental node from transmitted radiation is:

$$\Phi_{eTt} = \bar{\Phi}_{eT} + F \, \widetilde{\Phi}_{teT(t-\omega)} \qquad (A11.3)$$

$$\widetilde{\Phi}_{eTt} = \Phi_{eTt} - \bar{\Phi}_{eT} \qquad (A11.4)$$

$$\bar{\Phi}_{eT} = (1/24)\sum_{t=1}^{t=24}\Phi_{eTt} \qquad (A11.5)$$

$$\Phi_{eTt} = T\,I_t \qquad (A11.4)$$

where Φ_{eTt} is the overall gain to the environmental node from transmitted radiation at time t.

However, in practice, direct and diffuse transmitted radiation must be treated separately.

The gain to the environmental node due to conduction and radiation from the inner surface of the glazing is:

$$\Phi_{eAt} = \sum_{j=1}^{j=n}(H_{ej}\,A_j\,I_t) \qquad (A11.7)$$

where A is the component of the radiation absorbed by the glass, subscript j denotes the number of the glazing element and n is the total number of glazing elements within the window system.

Thus the total gain to the environmental node is:

$$\Phi_{et} = \Phi_{eTt} + \Phi_{eAt} \qquad (A11.8)$$

Additionally, if there is an internal blind, the gain to the air node is:

$$\Phi_{aAt} = \sum_{j=1}^{j=n}(H_{aj}\,A_j\,I_t) \qquad (A11.9)$$

Thus the total gain to the air node is:

$$\Phi_{at} = \Phi_{aAt} \qquad (A11.10)$$

To simplify the calculation of these gains, solar gain factors are used. These are the ratios of the components of the gain to the incident solar radiation. The room load has both steady state and cyclic components and the space gains are to the environmental and, possibly, the air nodes. Additionally, the surface factor depends on the response time of the space. To calculate the solar gain factors, typical values are taken, as follows:

— for slow response space: $F = 0.5$; time delay = 2 h

— for a fast response space: $F = 0.8$; time delay = 1 h

The solar gain factors are as follows:

$$\bar{S}_e = \bar{\Phi}_e / \bar{I} \qquad (A11.11)$$

$$\widetilde{S}_{et} = \widetilde{\Phi}_{et} / \widetilde{I}_t \qquad (A11.12)$$

$$\bar{S}_a = \bar{\Phi}_a / \bar{I} \qquad (A11.13)$$

$$\widetilde{S}_{at} = \widetilde{\Phi}_{at} / \widetilde{I}_t \qquad (A11.14)$$

Solar gain factors for generic glass and blind combinations are given in Table 5.20 (repeated here as Table 5.A11.1). These have been calculated using banded solar radiation data for Kew (1959–1968) incident on a south-west facing vertical window (see chapter 2). The transmission (T), absorption (A) and reflection (R) components (for thermal

Table 5.A11.1 Solar gain factors and shading coefficients for generic glazing/blind combinations

Description (inside to outside)	Solar gain factor at environmental node†			Solar gain factor at air node		Shading coefficient, S_c	
	$\bar{S}_e$	$\widetilde{S}_{el}$	$\widetilde{S}_{eh}$	$\bar{S}_a$	$\widetilde{S}_a$	Shortwave	Longwave
Single glazing/blind combinations:							
— clear glass	0.76	0.66	0.50	—	—	0.91	0.05
— absorbing glass	0.61	0.54	0.44	—	—	0.53	0.19
— absorbing slats/clear	0.43	0.44	0.44	0.17	0.18	—	—
— reflecting slats/clear	0.35	0.32	0.31	0.12	0.12	—	—
— 'generic' blind/clear	0.34	0.33	0.29	0.11	0.11	—	—
Double glazing/blind combinations:							
— clear/clear	0.62	0.56	0.44	—	—	0.70	0.12
— clear/reflecting	0.36	0.32	0.26	—	—	0.37	0.08
— low emissivity/clear	0.62	0.57	0.46	—	—	0.62	0.18
— low emissivity/absorbing	0.43	0.38	0.32	—	—	0.36	0.15
— low emissivity/clear/'generic' blind	0.15	0.14	0.11	—	—	—	—
— absorbing slats/clear/clear	0.34	0.36	0.37	0.18	0.21	—	—
— absorbing slats/clear/reflecting	0.19	0.19	0.19	0.12	0.13	—	—
— absorbing slats/low emissivity/clear	0.33	0.35	0.35	0.21	0.23	—	—
— absorbing slats/low emissivity/absorbing	0.22	0.22	0.22	0.16	0.17	—	—
— reflecting slats/clear/clear	0.28	0.29	0.26	0.15	0.16	—	—
— reflecting slats/clear/reflecting	0.17	0.16	0.16	0.10	0.10	—	—
— reflecting slats/low emissivity/clear	0.28	0.27	0.26	0.18	0.20	—	—
— reflecting slats/low emissivity/absorbing	0.18	0.17	0.17	0.14	0.15	—	—
— 'generic' blind/low emissivity/clear	0.29	0.29	0.27	0.17	0.18	—	—
Triple glazing:							
— clear/clear/clear	0.52	0.49	0.40	—	—	0.55	0.17
— clear/clear/absorbing	0.37	0.35	0.29	—	—	0.33	0.15
— clear/clear/reflecting	0.30	0.28	0.23	—	—	0.30	0.09
— clear/low emissivity/clear	0.53	0.50	0.42	—	—	0.50	0.21

† For $\widetilde{S}_e$, subscripts 'l' and 'h' denote thermally 'lightweight' and 'heavyweight' buildings, respectively

Note: shading coefficients for windows with slatted blind or windows with inner blind are not given since these not compatible with the properties of plain glass

Table 5.A11.2 Transmission, absorption and reflection components and emissivities for generic glass and blind combinations

Description	Shortwave radiation (proportions of total)			Longwave emissivity	
	Transmitted	Reflected 1	Reflected 2	Surface 1	Surface 2
Glass:					
— clear	0.789	0.072	0.072	0.837	0.837
— low emissivity*	0.678	0.091	0.108	0.837	0.17
— absorbing	0.46	0.053	0.053	0.837	0.837
— reflecting (high performance)*	0.39	0.31	0.45	0.837	0.025
Slatted blind†:					
— reflecting	0.0	0.60	0.40	0.80	0.80
— absorbing	0.0	0.80	0.20	0.80	0.80
'Generic' blind	0.20	0.40	0.40	0.80	0.80

* Asymmetric glass properties

shortwave radiation) and emissivities (for thermal long-wave radiation) for the generic glass and blind types used in calculating the solar gain factors are given in Table 5.A11.2.

Effectively, solar gain factors can only be calculated by means of a computer program. The following sections describe the basis of the calculation procedure.

5.A11.3 Transmission, absorption and reflection for direct solar radiation

Clear glass

For clear glass the transmission, absorption and reflection (TAR) coefficients can be derived theoretically (Jones, 1980).

The angle of refraction is obtained from the angle of incidence using Snell's Law:

$$\zeta_r = \arcsin(\sin \zeta_i / \mu) \qquad \text{(A11.15)}$$

The reflected beams for radiation polarised parallel to and perpendicular to the plane of incidence are determined using Fresnel's formula:

$$r_{//} = \frac{\tan^2(\zeta_i - \zeta_r)}{\tan^2(\zeta_i + \zeta_r)} \qquad \text{(A11.16)}$$

$$r_\perp = \frac{\sin^2(\zeta_i - \zeta_r)}{\sin^2(\zeta_i + \zeta_r)} \qquad \text{(A11.17)}$$

As the angle of incidence approaches 0 (i.e. normal incidence):

$$\tan \zeta_i > \sin \zeta_i > \zeta_i \qquad \text{(A11.18)}$$

hence:

$$r_{//} > r_\perp > \frac{(\mu - 1)^2}{(\mu + 1)^2} \qquad \text{(A11.19)}$$

This is a useful result as it enables the calculation of the extinction coefficient (k) if the transmission at normal incidence (T_n) is known. The extinction coefficient is a non-linear function of the glass thickness (L) and is related to the transmission coefficient by:

$$T_n = \frac{(1 - r)^2 \exp(-k\,L)}{1 - r^2 \exp(-2\,k\,L)} \qquad \text{(A11.20)}$$

For the beam polarised parallel to the plane of incidence, the fraction of incident energy absorbed for each beam is calculated as follows:

$$a_{//} = 1 - \exp(-k\,L / \cos \zeta_r) \qquad \text{(A11.21)}$$

and similarly for the perpendicularly polarised beam.

The transmitted, absorbed and reflected coefficients are calculated separately for each beam (i.e. parallel and perpendicularly polarised) and the average taken to give the overall coefficients. For the beam polarised parallel to the plane of incidence:

$$T_{D//} = \frac{(1 - r)^2 (1 - a_{//})}{1 - r^2 (1 - a_{//})^2} \qquad \text{(A11.22)}$$

$$A_{D//} = \frac{a_{//} (1 - r) [1 + r (1 - a_{//})]}{1 - r^2 (1 - a_{//})^2} \qquad \text{(A11.23)}$$

$$R_{D//} = \frac{r (1 - r)^2 (1 - a_{//})}{1 - r^2 (1 - a_{//})} + r \qquad \text{(A11.24)}$$

and similarly for the perpendicularly polarised beam.

Therefore:

$$T_D = \tfrac{1}{2} (T_{D//} + T_{D\perp}) \qquad \text{(A11.25)}$$

and similarly for the absorption and reflection coefficients.

Note that since the transmitted, absorbed and reflected components add up to unity, only two need be calculated, the third being obtained by subtraction.

Reflecting and other glasses

The characteristics of such glasses differ from those for plain glass and therefore must be obtained from the manufacturers. If the characteristics are supplied as a graph of tar coefficients against angle of incidence, the appropriate values can be read-off directly or by curve-fitting techniques.

Slatted blinds

The analysis is the same for both horizontal and vertical slatted blinds. Radiation may be transmitted into a room by the following paths (Parmelee and Vild, 1953).

— *direct*: i.e. passes through the blind without touching any surface; may be zero

— *reflected* (1): i.e. passes through the blind after one reflection from the slat surface which is directly irradiated by the sun

— *reflected* (2): i.e. passes through the blind after undergoing any number of reflections, the final reflection being from the slat surface opposite the one directly illuminated by the sun

— *reflected* (3): i.e. passes through the blind after undergoing any number of reflections, the final reflection being from the one directly illuminated by the sun.

In order to calculate these components, up to five configuration factors are required, each of which depends on the blind geometry. The number of factors needed depends on whether all or only part of the slat is illuminated.

The amount of a slat that is illuminated (i.e. not shaded by the slat above it) depends on the geometry of the blind and the 'profile angle'.

The profile angle (β) is the angle that the direct radiation beam makes with the blind in a vertical plane perpendicular to the plane of the window. For horizontal slatted blinds on a vertical window, the profile angle is the vertical shadow angle:

$$\beta = \sigma_v = \arctan (\tan h \sec \gamma_s) \qquad (A11.26)$$

For vertical slatted blinds on a vertical window, the profile angle is the wall–solar azimuth:

$$\beta = \gamma_s = \phi - \gamma \qquad (A11.27)$$

In the following analysis, it is assumed that the radiation is incident on the upper surface of the slat. The width of slat that is illuminated is calculated from:

$$M = \min \left(W, \frac{D \cos \beta}{\sin (\beta + \psi)} \right) \qquad (A11.28)$$

The configuration factors are calculated as follows.

Radiation that is reflected by the lower slat and passes into the room when the whole width is illuminated (C_1):

$$C_1 = \tfrac{1}{2} \{1 + (D / W) - [1 + (D^2 / W^2) + (2 D \sin \psi / W)]^{1/2}\} \qquad (A11.29)$$

Radiation that is reflected by the lower slat and intercepted by the upper slat when the whole width is illuminated (C2):

$$C_2 = \tfrac{1}{2} \{[1 + (D^2 / W^2) + (2 D \sin \psi / W)]^{1/2} + [1 + (D^2 / W^2) - (2 D \sin \psi / W)]^{1/2} - (2 D / W)\} \qquad (A11.30)$$

Radiation reflected by the upper slat which passes into the room (C_3):

$$C_3 = \tfrac{1}{2} \{[1 + (D / W) - [1 + (D^2 / W^2) - (2 D \sin \psi / W)]^{1/2}\} \qquad (A11.31)$$

Radiation reflected by the lower slat, which passes into the room when the lower slat is partially shaded (C_4):

$$C_4 = \tfrac{1}{2} (1 + \{[(W - M)^2 / M^2] + (D^2 / M^2) + [2 (W - M) D \sin \psi / \underline{M^2}]\}^{1/2} - [(W^2 / M^2) + (D^2 / M^2) + (2 W D \sin \psi / M^2)]^{1/2}) \qquad (A11.32)$$

Radiation reflected by the lower slat, which is intercepted by the upper slat when the lower slat is partially shaded (C_5):

$$C_5 = \tfrac{1}{2} ([(W^2 / M^2) + (D^2 / M^2) + (2 D W \sin \psi / M^2)]^{1/2} - (D / M) + [1 + (D^2 / M^2) - (2 D \sin \psi / M)]^{1/2} - \{[(W - M)^2 / M^2] + (D^2 / M^2) + [2 (W - M) D \sin \psi / M^2]\}^{1/2}) \qquad (A11.33)$$

If the whole of the lower slat is illuminated and some radiation may pass directly into the room, the TAR coefficients for the blind are calculated from:

$$T_D = 1 - \left(\frac{W \sin (\phi + \psi)}{D \cos \phi} \right) \times \left(1 - C_1 (1 - a) - \frac{C_2 (1 - a)^2 - [C_3 + C_1 C_2 (1 - a)]}{1 - C_2^2 (1 - a)^2} \right) \qquad (A11.34)$$

$$A_D = \frac{a W \sin (\phi + \psi)}{D \cos \phi [1 - C_2 (1 - a)]} \qquad (A11.35)$$

$$R_D = 1 - A_D - T_D \qquad (A11.36)$$

Where part of the lower slat is shaded by the slat above:

$$T_D = C_4 (1-a) + \{C_5 (1-a)^2$$

$$\times \left(\frac{C_3 + C_1 C_2 (1-a)}{1 - C_2^2 (1-a)^2} \right) \qquad (A11.37)$$

$$A_D = a \left(1 + \frac{C_5 (1-a)}{1 - C_2 (1-a)} \right) \qquad (A11.38)$$

$$R_D = 1 - A_D - T_D \qquad (A11.39)$$

Roller blinds

The properties for roller blinds are not well defined. It is generally sufficient to assume that the TAR coefficients are independent of the angle of incidence and take the values at normal incidence supplied by the manufacturers.

5.A11.4 Transmission, absorption and reflection for sky diffuse and ground reflected radiation

Transmission, absorption and reflection coefficients for glasses and blinds are calculated by considering the direct properties over a range of angles appropriate to the radiation. For glass, the TAR values for sky diffuse and ground reflected radiation are the same since glass has symmetrical properties. The characteristics for roller blinds can be assumed to be the same for direct and diffuse radiation. However, slatted blinds are highly asymmetrical so the two sources of diffuse radiation must be calculated separately.

Glasses

The standard properties are calculated on the assumption that the glass is exposed to a hemispherical source of uniform radiance therefore the transmission and absorption angles are from 0° to 90°. Mathematically, the expressions for tar could be integrated over this range, i.e.:

$$T_d = \int_0^{90} T_d \, \xi_i \sin (2 \, \xi_i) \, d\xi_i \qquad (A11.40)$$

In practice the direct properties are summed for angles of incidence from 2.5° to 87.5° at intervals of 5°, i.e.:

$$T_d = \sum_{\zeta=2.5}^{\zeta=87.5} \{T_D \, \zeta \, [\sin^2 (\xi_i + 2.5) - \sin^2 (\xi_i - 2.5)]\} \qquad (A11.41)$$

A_d is calculated similarly and R_d is obtained by subtraction from unity, see equation A11.36.

Slatted blinds

The direct properties are summed for profile angles from 5° to 85° at intervals of 10° for sky diffuse radiation. For ground reflected radiation, they are summed from –85° to –5° at intervals of 10° taking into account the configuration factor of the hemispherical radiating source bounded by profile angles of $(\beta + 5)°$ and $(\beta - 5)°$ (Nicol, 1966). Thus, for sky diffuse radiation:

$$T_{ds} = \sum_{\beta=5}^{\beta=85} \{T_D \, \beta \, [\sin (\beta + 2.5) - \sin (\beta - 2.5)]\} \qquad (A11.42)$$

For ground reflected radiation:

$$T_{dg} = \sum_{\beta=-5}^{\beta=-85} \{T_D \, \beta \, [\sin (\beta + 2.5) - \sin (\beta - 2.5)]\} \qquad (A11.43)$$

A_{ds} and A_{dg} are calculated similarly and R_{ds} and R_{dg} are obtained by subtraction from unity, see equation A11.36.

5.A11.5 Properties of glass and blind combinations

The properties of multiple layer windows can be calculated from the properties of the individual components. There are many glass types and many permutations; the method of calculation is demonstrated in the following for double and triple glazing using generic glass and blind types.

In the same way that the properties of a single sheet of glass are calculated from the fundamental properties of the glass and an infinite number of inter-reflections at both glass/air interfaces, the properties of multiple glazing are calculated by considering the inter-reflections between the component layers (Jones, 1980; Mitalas and Stephenson, 1962). These calculations are performed for both direct and diffuse radiation. However, if the window incorporates a blind, the radiation reflected by or transmitted through it is assumed to be diffuse whatever the nature of the source. This is because the slat surfaces are assumed to be diffusing rather than specular reflectors (Parmelee and Vild, 1953).

The following equations are derived from Figure 5.A11.11 where all layers are symmetrical, i.e. both surfaces of the layer have the same reflection and the specularity of the radiation is not changed by the layer. If any of the layers are asymmetrical, the equations become more complicated since they have to include the reflection of both surfaces of the layer. If any of the layers is a diffusing slatted blind, then the direct radiation equations need to include the diffuse properties of the elements for radiation that has been reflected by the blind(s). Examples for some of these situations are given elsewhere (Jones, 1980).

Double glazing

TAR coefficients for double glazing, denoted by prime (′), are as follows:

$$T' = (T_o \, T_i) / (1 - R_o \, R_i) \qquad (A11.44)$$

$$A_o' = A_o + [(T_o \, A_o \, R_i) / (1 - R_o \, Ri)] \qquad (A11.45)$$

$$A_i' = (T_o \, A_i) / (1 - R_o \, R_i) \qquad (A11.46)$$

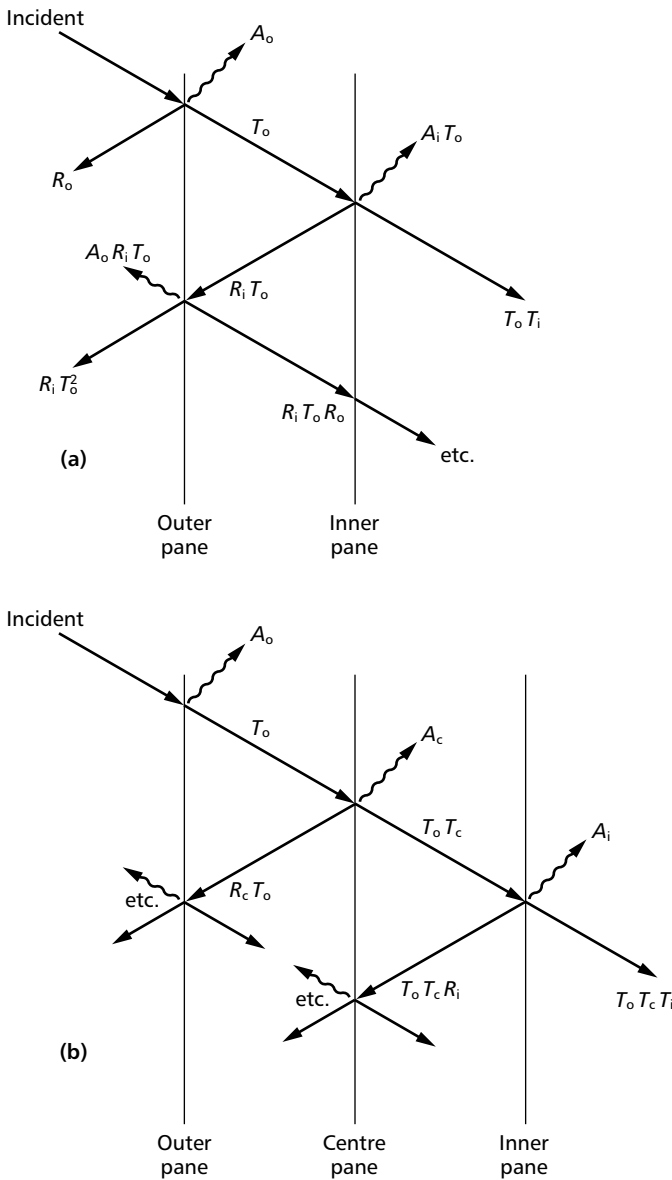$$R' = 1 - T' - A_o' - A_i' \qquad (A11.47)$$

Figure 5.A11.11 Transmitted, absorbed and reflected radiation; (a) double glazing, (b) triple glazing

where subscript 'o' denotes the outer glazing element and subscript 'i' denotes the inner glazing element.

Triple glazing

TAR coefficients for triple glazing, denoted by double prime (″), are as follows:

$$T'' = \frac{T_o T_c T_i}{(1 - R_o R_c)(1 - R_c R_i) - T_c^2 R_o R_i} \quad \text{(A11.48)}$$

$$A_o'' = A_o + \frac{T_o A_o R_c}{1 - R_o R_c}$$
$$+ \frac{T_o T_c^2 A_o R_i}{(1 - R_o R_c)(1 - R_c R_i) - T_c^2 R_o R_i} \quad \text{(A11.49)}$$

$$A_c'' = \frac{T_o A_c (1 - R_c R_i + T_c R_i)}{(1 - R_o R_c)(1 - R_c R_i) - T_c^2 R_o R_i} \quad \text{(A11.50)}$$

$$A_i'' = \frac{T_o T_c A_i}{(1 - R_o R_c)(1 - R_c R_i) - T_c^2 R_o R_i} \quad \text{(A11.51)}$$

$$R'' = 1 - T'' - A_o'' - A_c'' - A_i'' \quad \text{(A11.52)}$$

where subscript 'o' denotes the outer glazing element, subscript 'c' denotes the central glazing element and subscript 'i' denotes the inner glazing element.

The heat gain to the environmental node due to conduction and radiation from the inner surface of the glazing is given by equation A11.7. If there is an internal blind, the additional heat gain to the air node is given by equation A11.9. In these equations, the transmittance factors (H) depend on the values taken for the thermal resistances (i.e. the radiant and convective heat transfer coefficients) of the layers of the window. They are calculated by considering the thermal resistance network for the window. Figure
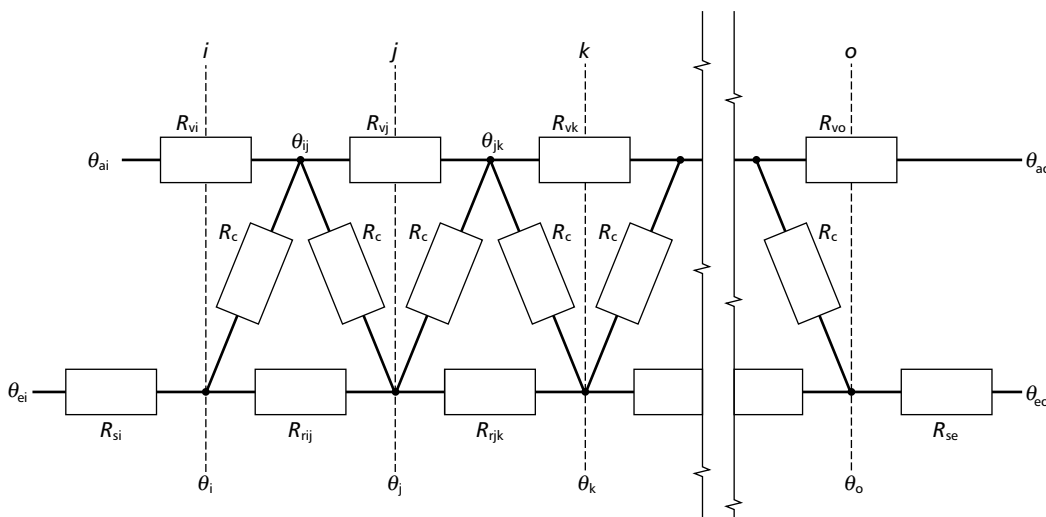

Figure 5.A11.12 General thermal resistance network for a multiple-layer window

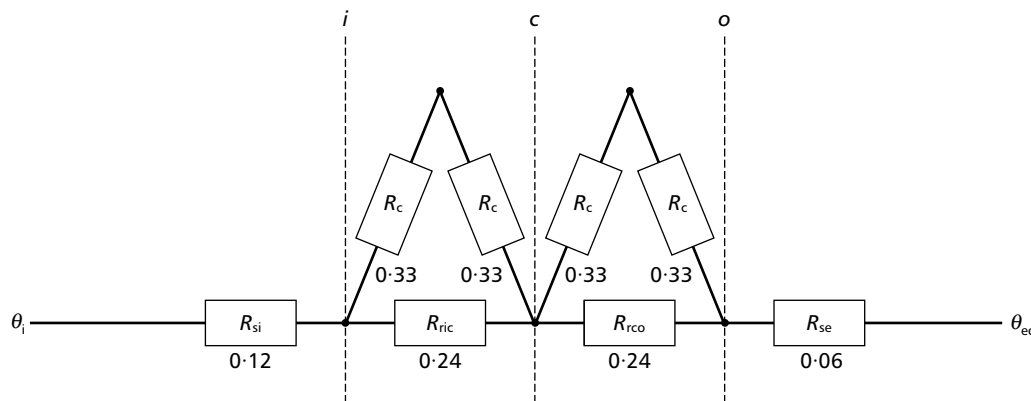5.A11.12 shows the general thermal resistance network for a multiple-layer window.

The properties of the glazing systems are calculated using the following standard thermal resistances and heat transfer coefficients:

— thermal resistance between inner surface of window and environmental point (i.e. inside thermal resistance): $R_{si} = 0.12$ m^2·K·W^{-1}

— thermal resistance between outer surface of window and sol-air temperature (i.e. outside thermal resistance): $R_{se} = 0.06$ m^2·K·W^{-1}

— convective resistance between a window layer and the air: $R_c = 0.33$ m^2·K·W^{-1} (for vertical window, corresponding to $h_c = 3$ W·m^2·K^{-1})

— radiative resistance between two layers (j, k) of window:

$$(R_r)_{j,k} = (\varepsilon_j + \varepsilon_k - \varepsilon_j\,\varepsilon_k) / (h_r\,\varepsilon_j\,\varepsilon_k) \qquad \text{(A11.53)}$$

(if both layers have an emissivity of 0.84 and $h_r = 5.7$ W·m^{-2}·K^{-1}, $(R_r)_{j,k} = 0.24$ m^2·K·W^{-1})

— ventilation resistance across window layer between adjacent air spaces: $R_v = 0$ m^2·K·W^{-1} if the layer is a blind; $R_v = \infty$ if the layer is glass.

Example A11.1: Triple glazing without blinds

Figure 5.A11.13 shows the network for triple glazing and Figure 5.A11.14 shows the simplified network resulting from evaluation of the parallel resistances.

The total resistance of the network is:

$$\Sigma R = R_{si} + R_{ic} + R_{co} + R_{se}$$

$$= 0.12 + 0.18 + 0.18 + 0.06 = 0.54 \text{ m}^2\text{·K·W}^{-1}$$

where R_{ic} is the thermal resistance between inner and central elements of the glazing (m^2·K·W^{-1}) and R_{co} is the thermal resistance between central and outer elements of the glazing (m^2·K·W^{-1}).

The transmittance factors for the inner, central and outer elements of the glazing can be shown to be:

$$H_{ei} = (R_{ic} + R_{co} + R_{se}) / \Sigma R$$

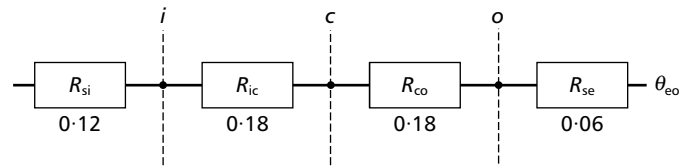$$= (0.18 + 0.18 + 0.06) / 0.54 = 0.78$$

$$H_{ec} = (R_{co} + R_{se}) / \Sigma R$$

$$= (0.18 + 0.06) / 0.54 = 0.44$$

$$H_{eo} = R_{se} / \Sigma R = 0.06 / 0.54 = 0.11$$

From equation A11.7, the cyclic component of the convective and longwave radiant gain from the glazing to the environmental node is calculated as follows:

$$\widetilde{H}_e A = H_{ei}\widetilde{A}_i + H_{ec}\widetilde{A}_c + H_{eo}\widetilde{A}_o$$

Figure 5.A11.14 Simplified thermal resistance network for triple glazing

Figure 5.A11.13 Thermal resistance network for triple glazing

Table 5.A11.3 Example A11.1: components of radiation

Time / h	Solar irradiance / W·m^{-2}	Radiation absorbed/transmitted by glazing system / W·m^{-2}				Gains to space / W·m^{-2}		
		Radiation absorbed by inner, central and outer glazing elements			Directly transmitted radiation, T	Cyclic component of absorbed radiation, $(H_e A)$	Cyclic component of transmitted radiation for lightweight (L) and heavyweight (H) buildings	
		A_i	A_c	A_o			$\widetilde{T}_L$	$\widetilde{T}_H$
1200	442	35	58	89	133	37	—	—
1300	531	46	71	104	189	53	54	—
1400	572	52	77	108	223	60	99	34
1500	563	52	75	105	229	59	126	62
1600	504	47	67	94	205	50	131	79
Mean:	179	15	24	34	65	26	—	—

Table 5.A11.3 summarises the steps in the calculation of the solar gain to the space by means of an example. The calculation was carried out as follows.

For 12:00 h:

$$\widetilde{H}_e A = 0.78\,(35 - 15) + 0.44\,(58 - 24)$$

$$+ 0.11\,(89 - 34) = 36.6$$

The gain at other times is calculated similarly. The mean gain is calculated using the mean, rather than the cyclic, absorption values.

The cyclic component of the directly transmitted short-wave radiation is attenuated by the surface factor (F), which is appropriate to the thermal weight of the building and corresponding time delay, see Table 5.A11.4.

For a lightweight building, $F = 0.8$ and the time delay is 1 hour, i.e.:

$$\widetilde{T}_L = 0.8 \times (T_{t+1} - \bar{T})$$

and for a heavyweight building, $F = 0.5$ and the time delay is 2 hours, i.e.:

$$\widetilde{T}_H = 0.5 \times (T_{t+2} - \bar{T})$$

where subscript 'L' denotes thermally lightweight building and subscript 'H' denotes thermally heavyweight building.

The mean solar gain factor is given by:

$$\bar{S}_e = \frac{\text{mean transmitted radiation plus mean absorbed radiation}}{\text{daily mean incident radiation}}$$

Hence:

$$\bar{S}_e = (65 + 26)\,/\,179 = 0.51$$

The cyclic solar gain factors are calculated using the gains appropriate to a time one or two hours after the time of peak radiation, depending on the thermal weight of the structure, i.e.:

$$\widetilde{S}_e = \frac{\text{total swing in gain to space}}{\text{swing in external gain}}$$

Peak solar irradiance occurs at 14:00 h; hence, for a thermally lightweight structure (i.e. 1-hour delay):

$$\widetilde{S}_{eL} = (126 + 59)\,/\,(563 - 179) = 0.48$$

and for a thermally heavyweight structure (i.e. 2-hour delay):

$$\widetilde{S}_{eH} = (79 + 50)\,/\,(504 - 179) = 0.4$$

Example A11.2: Single glazing with internal absorbing blind

Figure 5.A11.15 shows the network for single glazing with an internal blind and Figure 5.A11.16 shows the simplified network resulting from evaluation of the parallel resistances.

In this case, there are transmittance factors to both the air and environmental nodes, which are calculated as follows:

$$R_{ix} = R_{rio} + [(R_c R_{se})\,/\,(R_c + R_{se})]$$

$$= 0.23 + [(0.33 \times 0.06)\,/\,(0.33 + 0.06)] = 0.28$$

$$H_{ei} = \frac{R_c R_{ix}\,/\,(R_c + R_{ix})}{R_{si} + [(R_c R_{ix})\,/\,(R_c + R_{ix})]}$$

$$= \frac{(0.33 \times 0.28)\,/\,(0.33 + 0.28)}{0.12 + [(0.33 \times 0.28)\,/\,(0.33 + 0.28)]} = 0.56$$

$$H_{ai} = \frac{R_{si} R_{ix}\,/\,(R_{si} + R_{ix})}{R_c + [(R_{si} R_{ix})\,/\,(R_{si} + R_{ix})]}$$

$$+ \left(\frac{R_{si} R_c\,/\,(R_{si} + R_c)}{R_{ix} + [(R_{si} R_c)\,/\,(R_{si} + R_c)]}\right)\left(\frac{R_{se}}{R_{se} + R_c}\right)$$

$$= 0.24$$

$$R_{ox} = R_{rio} + [(R_c R_{si})\,/\,(R_c + R_{si})] = 0.32$$

$$H_{eo} = \frac{R_c R_{se}\,/\,(R_c + R_{se})}{R_{ox} + (R_c R_{se})\,/\,(R_c + R_{se})} \times \frac{R_c}{(R_c + R_{si})}$$

$$= 0.10$$

$$H_{ao} = \frac{R_{se} R_{ox}\,/\,(R_{se} + R_{ox})}{R_c + [(R_{se} R_{ox})\,/\,(R_{se} + R_{ox})]}$$

$$+ \left(\frac{R_{se} R_c\,/\,(R_{se} + R_c)}{R_{ox} + [(R_{se} R_c)\,/\,(R_{se} + R_c)]}\right)\left(\frac{R_{si}}{R_c + R_{si}}\right)$$

$$= 0.17$$

Table 5.A11.4 Thermal response

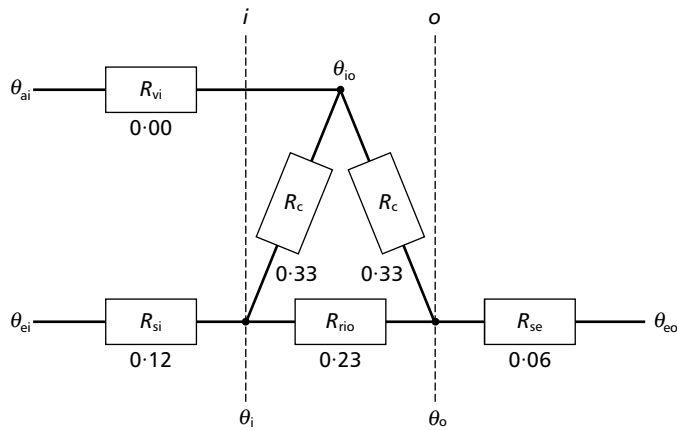| Thermal response	Typical features of construction	Response factor, f_r	Response to short-wave radiation		Time lead for admittance, $\omega\,/\mathrm{h}$
			Average surface factor, F	Time delay, $\phi\,/\,\mathrm{h}$	
Slow	Masonry external walls and Internal partitions, bare solid floors and ceilings	> 4	0.5	2	1
Fast	Lightweight external cladding, de-mountable partitions, suspended ceilings, solid floors with carpet or wood block finish or suspended floors	≤ 4	0.8	1	0

Figure 5.A11.15 Thermal resistance network for single glazing with internal blind
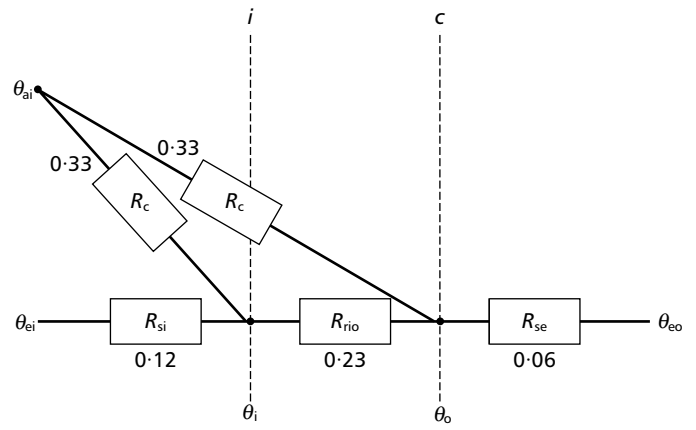

Figure 5.A11.16 Simplified thermal resistance network for single glazing with internal blind

Table 5.A11.5 Example A11.2: components of radiation

Time (h)	Solar irradiance / W·m^{-2}	Radiation absorbed/transmitted by glazing system / W·m^{-2}			Gains to space / W·m^{-2}			
		Radiation absorbed by inner, and outer glazing elements		Directly transmitted radiation, T	Cyclic components of absorbed radiation		Cyclic components of transmitted radiation for lightweight (L) and heavyweight (H) buildings	
		A_i	A_o		H_{eA}	H_{aA}	$\widetilde{T}_L$	$\widetilde{T}_H$
1200	442	116	146	47	45	53	—	—
1300	531	151	178	52	68	67	22	—
1400	572	171	191	54	81	74	26	14
1500	563	173	186	51	81	73	28	17
1600	504	157	167	45	70	66	26	18
Mean:	179	51	58	19	34	22	—	—

Table 5.A11.5 summarises the steps in the calculation of the solar gain to the space.

The cyclic component of the convective and longwave radiant gain from the glazing to the environmental node is calculated as for triple glazing, see example A11.1, i.e.:

$$\widetilde{H}_e A = H_{ei} \widetilde{A}_i + H_{ec} \widetilde{A}_c + H_{eo} \widetilde{A}_o$$

The instantaneous component of the convective and long-wave radiant gain from the glazing to the air node is calculated as follows:

$$H_a A = H_{ai} A_i + H_{ao} A_o$$

Hence, for 12:00 h:

$$H_a A = (0.24 \times 116) + (0.17 \times 146) = 52.66$$

The gain at other times is calculated similarly, see Table 5.A11.5. The mean gain is calculated using the mean, rather than the cyclic, absorption values.

The cyclic component of the directly transmitted short-wave radiation and attenuated by the surface factor (F) appropriate to the thermal weight of the building and delayed by a time corresponding to the thermal weight, see Table 5.A11.4.

For a lightweight building, $F = 0.8$ and the time delay is one hour, i.e.:

$$\widetilde{T}_L = 0.8 \times (T_{t+1} - \overline{T})$$

For a heavyweight building, $F = 0.5$ and the time delay is two hours, i.e.:

$$\widetilde{T}_H = 0.5 \times (T_{t+2} - \overline{T})$$

where subscript 'L' denotes thermally lightweight building and subscript 'H' denotes thermally heavyweight building.

As for example A11.1, the mean solar gain factor at the environmental node is given by:

$$\overline{S}_e = \frac{\text{mean transmitted radiation plus mean absorbed radiation}}{\text{daily mean incident radiation}}$$

Hence:

$$\overline{S}_e = (19 + 34) / 179 = 0.3$$

Again, as for example A11.1, the cyclic solar gain factors at the environmental node are calculated using the gains appropriate to a time depending on the thermal weight of the structure, i.e.:

$$\widetilde{S}_e = \frac{\text{total swing in gain to space}}{\text{swing in external gain}}$$

Peak solar irradiance occurs at 14:00; hence, for a thermally lightweight structure (i.e. 1-hour delay):

$$\widetilde{S}_{eL} = (28 + 81) / (563 - 179) = 0.28$$

and for a thermally heavyweight structure (i.e. 2-hour delay):

$$\widetilde{S}_{eH} = (18 + 70) / (504 - 179) = 0.27$$

The mean solar gain factor at the air node is given by:

$$\bar{S}_a = \frac{\text{mean gain to air node}}{\text{mean incident radiation}}$$

Hence:

$$\bar{S}_a = 22 / 179 = 0.12$$

The cyclic solar gain factor at the air node is given by:

$$\widetilde{S}_a = \frac{\text{total swing in gain to air node}}{\text{swing in external gain}}$$

There is no time delay associated with the air node, hence:

$$\widetilde{S}_a = 74 / (572 - 179) = 0.19$$

5.A11.6   Shading coefficients

In addition to solar gain factors, Table 5.A11.1 also gives the shortwave and longwave shading coefficients (S_c). These correspond to the direct and indirect transmission to the space for direct radiation at normal incidence (see 5.6.2) divided by (0.87 × total incident radiation), where 0.87 is transmission coefficient of nominal 4 mm plain glass at normal incidence. However, it should be noted that the room gains, and hence the solar gain factors, depend on the direct and diffuse components of the incident radiation and the angle of incidence of the direct radiation.

The *G*-value does not distinguish between the two components of the transmission nor make reference to clear glass. Numerically, therefore, it is equal to the sum of the shortwave and longwave shading coefficients multiplied by 0.87.

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