

## Appendix 5.A7: Derivation of factor for intermittent heating

The symbols used in this appendix are defined in section 5.2.1.

Rearranging equation 5.44 provides the definition of the factor for intermittent heating,  $F_3$ :

$$F_3 = \Phi_i / \Phi_t \quad (\text{A7.1})$$

where  $F_3$  is a correction factor for intermittent heating,  $\Phi_i$  is the plant size for intermittent operation (W), and  $\Phi_t$  the total heat loss (W).

Assuming that the installed capacity is that required to raise the space temperature from the daily mean space temperature ( $\bar{\theta}_i$ ) to the internal design temperature ( $\theta_i$ ) then, for a design day where the mean outside temperature ( $\bar{\theta}_o$ ) is equal to the design outside temperature:

$$F_3 = \frac{[\Sigma(AU) + C_v](\bar{\theta}_i - \bar{\theta}_o) + [\Sigma(AY) + C_v](\theta_i - \bar{\theta}_i)}{[\Sigma(AU) + C_v](\theta_i - \bar{\theta}_o)} \quad (\text{A7.2})$$

Assuming the ventilation rate is constant and equal to the design value:

$$F_3 = \frac{\bar{\theta}_i - \bar{\theta}_o}{\theta_i - \bar{\theta}_o} + \frac{[\Sigma(AY) + C_v](\theta_i - \bar{\theta}_i)}{[\Sigma(AU) + C_v](\theta_i - \bar{\theta}_o)} \quad (\text{A7.3})$$

$$F_3 = \frac{\bar{\theta}_i - \bar{\theta}_o}{\theta_i - \bar{\theta}_o} + f_r \left( \frac{\theta_i - \bar{\theta}_i}{\theta_i - \bar{\theta}_o} \right) \quad (\text{A7.4})$$

where  $f_r$  is the thermal response factor (see equation 5.14).

It has been shown (Harrington-Lynn, 1998) that:

$$\frac{\bar{\theta}_i - \bar{\theta}_o}{\theta_i - \bar{\theta}_o} = \frac{Hf_r}{Hf_r + (24 - H)} \quad (\text{A7.5})$$

where  $H$  is hours of plant operation including preheat (h).

Therefore, subtracting both sides from  $\theta_i$  and rearranging gives:

$$\frac{\theta_i - \bar{\theta}_i}{\theta_i - \bar{\theta}_o} = 1 - \frac{Hf_r}{Hf_r + (24 - H)} \quad (\text{A7.6})$$

Substituting equations A7.6 and A7.5 into equation A7.4 gives:

$$F_3 = \frac{24f_r}{Hf_r + (24 - H)} \quad (\text{A7.7})$$

### Reference for Appendix 5.A8

Harrington-Lynn J (1998) 'Derivation of equations for intermittent heating used in CIBSE Building Energy Code Part 2a' *Building Serv. Eng. Res. Technol.* 19(4)

## Appendix 5.A8: Derivation of thermal steady state model

### 5.A8.1 Notation

Symbols used in this appendix are as follows.

$A_n$	Area of surface $n$ ( $\text{m}^2$ )
$a, b$	Linearising constants
$b_n$	Radiant heat transfer coefficient ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$c_p$	Specific heat capacity of air ( $\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$ )
$c_v$	Ventilation conductance ( $\text{W}\cdot\text{K}^{-1}$ )
$E_{bn}$	Black body radiation from surface $n$ ( $\text{W}\cdot\text{m}^{-2}$ )
$F_a$	fraction of air temperature detected by sensor (0.5 for a sensor detecting operative temperature)
$F_{m,n}$	View factor from surface $m$ to surface $n$
$h_a$	Heat transfer coefficient between air and environmental nodes ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$H_c$	Thermal transmittance due to convection ( $\text{W}\cdot\text{K}^{-1}$ )
$h_c$	Convective heat transfer coefficient ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$h_{cn}$	Convective heat transfer coefficient for surface $n$ ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$H_r$	Thermal transmittance due to radiation ( $\text{W}\cdot\text{K}^{-1}$ )
$h_r$	Radiative heat transfer coefficient ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$\mathcal{J}_n$	Radiosity of surface $n$ ( $\text{W}\cdot\text{m}^{-2}$ )
$L_n$	Longwave radiant heat flux incident on surface $n$ ( $\text{W}\cdot\text{m}^{-2}$ )
$m$	Integer denoting particular surface
$\dot{m}_a$	Mass flow rate of air ( $\text{kg}\cdot\text{s}^{-1}$ )
$N$	Total number of surfaces
$N_v$	Number of room air changes (h <sup>-1</sup> )
$n$	Integer denoting particular surface
$R$	Radiant fraction of source from source
$R_{sin}$	Thermal resistance between inner face of surface $n$ and environmental temperature ( $\text{m}^2\cdot\text{K}\cdot\text{W}^{-1}$ )
$R_{sn}$	Thermal resistance of surface $n$ ( $\text{m}^2\cdot\text{K}\cdot\text{W}^{-1}$ )
$U_n$	Thermal transmittance for material of which surface $n$ is composed ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$U_n'$	Thermal transmittance between inner face of surface $n$ and heat transfer temperature on outer face of surface $n$ ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$U_p$	Thermal transmittance modified for heat flow through internal partition ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$V$	Room volume ( $\text{m}^3$ )
$\alpha$	Surface absorption coefficient
$\varepsilon_n$	Emissivity of surface $n$
$\Phi_{con}$	Convective energy from emitter (W)
$\Phi_f$	Fabric heat gain (W)
$\Phi_{ln}$	Longwave energy incident on surface $n$ from sources other than room surfaces (W)
$\Phi_{rad}$	Radiant energy from emitter (W)
$\Phi_t$	Total heat loss (W)
$\phi_n$	Radiant heat flow from surface $n$ ( $\text{W}\cdot\text{m}^{-2}$ )
$\theta_{ai}$	Inside air temperature ( $^{\circ}\text{C}$ )
$\theta_{ain}$	Air temperature for convective heat exchange with surface $n$ ( $^{\circ}\text{C}$ )
$\theta_c$	Operative temperature at centre of room ( $^{\circ}\text{C}$ )
$\theta_c'$	Operative temperature on far side of internal partition through which heat flow occurs ( $^{\circ}\text{C}$ )
$\theta_{ei}$	Environmental temperature ( $^{\circ}\text{C}$ )
$\theta_o$	External heat transfer temperature ( $^{\circ}\text{C}$ )
$\theta_{on}$	External heat transfer temperature for surface $n$ ( $^{\circ}\text{C}$ )
$\theta_r$	Mean radiant temperature ( $^{\circ}\text{C}$ )
$\theta_s$	Surface temperature ( $^{\circ}\text{C}$ )
$\theta_{sn}$	Surface temperature of surface $n$ ( $^{\circ}\text{C}$ )
$\theta^*$	Radiant-star temperature ( $^{\circ}\text{C}$ )

### 5.A8.2 Full model

The rate of loss of heat from a space through the building fabric can be expressed as:

$$\Phi_f = \sum_{n=1}^N A_n (\theta_{sn} - \theta_{on}) / R_{sn} \quad (\text{A8.1})$$

The rate of heat flow through a wall is equal to that into the wall, thus the fabric heat loss can also be expressed as:

$$\Phi_f = \sum_{n=1}^N [A_n (\theta_{ain} - \theta_{sn}) h_{cn} - \phi_n] \quad (\text{A8.2})$$

Note that  $\phi_n$  is positive for heat flows leaving the surface.

The first term inside the square brackets represents the rate of convection of heat from the room air to the surface and the second term is the rate of radiant heat flow into the surface. This radiant term represents the exchange of longwave radiation between the surface and all other surfaces within the room. (The calculation of steady state loss ignores shortwave radiation.) The exchange of longwave radiation can be seen as analogous to the reflection of light from a diffuse source, i.e. there are an infinite number of reflections of radiation between the surfaces.

The rate of radiant heat flow into the surface is the difference between that incident ( $L_n$ ) upon the surface and that leaving the surface ( $\mathcal{J}_n$ ) that is:

$$\phi_n = A_n (\mathcal{J}_n - L_n) \quad (\text{A8.3})$$

Now the rate at which radiant energy leaves a surface may be expressed as:

$$\mathcal{J}_n = (1 - \varepsilon_n) L_n + \varepsilon_n E_{bn} \quad (\text{A8.4})$$

Thus:

$$\phi_n = A_n (E_{bn} - \mathcal{J}_n) \varepsilon_n / (1 - \varepsilon_n) \quad (\text{A8.5})$$

The radiation incident upon the surface is the sum of that received from other surfaces and that from radiant heating sources. Radiation from another surface depends upon the view factor between that surface and the subject surface ( $n$ ) and the rate at which radiation leaves that surface (i.e. the radiosity).

Thus the radiation incident upon surface  $n$  is:

$$A_n L_n = \sum_{m=1}^N (\mathcal{J}_m A_m F_{m,n}) - \Phi_{ln} \quad (\text{A8.6})$$

However:

$$A_n F_{n,m} = A_m F_{m,n} \quad (\text{A8.7})$$

Therefore:

$$A_n L_n = \sum_{m=1}^N (\mathcal{F}_m A_n F_{n,m}) - \Phi_n \quad (\text{A8.8})$$

This represents a set of simultaneous equations (one for each surface) that when solved give the amount of radiation leaving each surface ( $\mathcal{F}_n$ ). Simultaneous solution means that the infinite number of reflections of radiation is accounted for automatically.

In order to solve equation A8.8 it is first necessary to substitute for  $L_n$  by combining equations A8.5 and A8.8. Assuming that  $F_{n,m}$  is zero (i.e. that is all surfaces are planar), this results in the following equation set:

$$\begin{aligned} \mathcal{F}_n / (1 - \varepsilon_n) - \sum_{m=1}^N F_{n,m} \mathcal{F}_m \dots \\ = \varepsilon_n E_{bn} / (1 - \varepsilon_n) + \Phi_n / A_n \end{aligned} \quad (\text{A8.9})$$

This relationship is converted into a heat loss model by linearising the black body emissive power and introducing the steady state surface heat balance. Thus:

$$E_{bn} = a + b \theta_{sn} \quad (\text{A8.10})$$

where  $a$  and  $b$  are constants.

From equation A8.1, for a single surface ( $n$ ):

$$\Phi_f = A_n (\theta_{sn} - \theta_{on}) U_n' \quad (\text{A8.11})$$

Therefore, equating A8.11 and A8.2 to eliminate  $\Phi_f$  gives:

$$-\phi_n + \theta_{ain} h_{cn} + U_n' \theta_{on} = \theta_{sn} (h_{cn} + U_n') \quad (\text{A8.12})$$

$U_n'$  is the transmittance between the surface temperature  $\theta_{sn}$  and the outside temperature  $\theta_{on}$ , i.e. the heat transfer temperature on the other side surface  $n$ , given by:

$$U_n' = U_n / (1 - U_n R_{sin}) \quad (\text{A8.13})$$

$R_{sin}$  is the standard value of the inner surface resistance used to calculate the standard  $U$ -value (see chapter 3 of this Guide) for surface  $n$ ,  $U_n$ . Since  $U_n'$  is also dependent on the external surface heat transfer coefficient, i.e. the surface coefficient appropriate to the 'other' side of surface, it may be necessary to include a correction for exposure.

Substitution of equations A8.10 and A8.12 into equation A8.9 gives the set of equations A8.14 (see below), which represent both radiant interchange between surfaces and the conduction of heat through room surfaces.

Equation A8.14 places no restrictions on the air temperature distribution within the space. A means of obtaining air

temperatures would be to combine the solution of the above with computational fluid dynamics. Alternatively, some rules could be assigned to the distribution of air temperature throughout the space (Gagneau et al., 1997).

### 5.A8.3 Reference model

The reference model is developed by adding convective heat transfer and control sensor models to the full model and making some assumptions about the distribution of the radiant component of heat from the emitter.

The full model contains an arbitrary model of the convective heat transfer process. The reference model assumes a fully mixed space, i.e. the dry bulb temperature of the air does not vary from point to point within the space. Thus in equation A8.14 all values of  $\theta_{ain}$  are equal to the inside air temperature  $\theta_{ai}$  and the convective heat balance is then given by:

$$\begin{aligned} - \sum_{n=1}^N h_{cn} A_n \theta_{sn} + \theta_{ai} (\dot{m}_a c_p + \sum_{n=1}^N h_{cn} A_n) \\ = \Phi_t (1 - R) + \theta_{ao} \dot{m}_a c_p \end{aligned} \quad (\text{A8.16})$$

The model is completed by the introduction of the control temperature ( $\theta_c$ ), for example the operative temperature which at low air speeds is the average of the air and mean radiant temperatures. The mean radiant temperature 'seen' by a sensor may be considered to be the equivalent temperature for radiant heat exchange between the sensor and its surroundings. It therefore depends upon:

- surface temperature
- surface emissivity
- emissivity of the sensor
- view factor between the surfaces and the sensor
- radiation from a heat emitter incident on the sensor.

Thus, the mean radiant temperature varies throughout the space. It is possible to model the sensor as an additional room surface. However, for the purposes of design calculations, the sensor is deemed to be located at a position where the proportion of longwave radiation received from each surface is directly proportional to the ratio of the area of the surface to the total room area. Furthermore, the sensor is also assumed to have an emissivity of unity (i.e. a black body). Thus the design mean radiant temperature is:

$$\theta_r = \frac{\sum \theta_{sn} A_n}{\sum A_n} + \frac{R \Phi_t}{h_r \sum A_n} \quad (\text{A8.17})$$

Note that  $h_r$  is calculated for an emissivity of unity.

$$\begin{aligned} (h_{cn} + U_n' + \varepsilon_n h_{rn}) (\theta_{sn} / \varepsilon_n) - \sum_{m=1}^N (F_{n,m} / \varepsilon_m) [(h_{cm} + U_m') (1 - \varepsilon_m) + h_r \varepsilon_m] \theta_{sm} - (h_{cn} \theta_{ain} / \varepsilon_n) + \sum_{m=1}^N (F_{n,m} / \varepsilon_m) [h_{cm} (1 - \varepsilon_m) \theta_{aim}] \\ = (\theta_{om} U_n' / \varepsilon_n) - \sum_{m=1}^N (F_{n,m} / \varepsilon_m) [U_m' (1 - \varepsilon_m) \theta_{om}] + \Phi_n / A_n \end{aligned} \quad (\text{A8.14})$$

where:

$$h_{rn} = \varepsilon_n b_n \quad (\text{A8.15})$$

The control temperature is given by:

$$\theta_c = F_a \theta_{ai} + (1 - F_a) \theta_r \quad (\text{A8.18})$$

where  $F_a = 0.5$  if the sensed parameter is the operative temperature.

Assuming that any radiant heat input is uniformly distributed over each surface, and is equal to  $(\Phi_r R / \Sigma A)$ , the reference model may be represented by the equation set:

$$\mathbf{A} \mathbf{X} = \mathbf{C} \quad (\text{A8.19})$$

where  $\mathbf{A}$ ,  $\mathbf{X}$  and  $\mathbf{C}$  are matrices, as defined in the following boxes.

Matrix  $\mathbf{A}$ :

(a) Surface heat balance equations

Terms  $A(n,n)$  for  $n = 1$  to  $n =$  total number of room surfaces:

$$A(n,n) = (h_{cn} + U_n' + h_r \varepsilon_n) / \varepsilon_n \quad (\text{A8.20})$$

Terms  $A(n,m)$  where  $n \neq m$ , for  $n = 1$  to  $n =$  total number of room surfaces and for  $m = 1$  to  $m =$  total number of room surfaces:

$$A(n,m) = -F_{n,m} [(h_{cm} + U_m') (1 - \varepsilon_m) + h_r \varepsilon_m] / \varepsilon_m \quad (\text{A8.21})$$

Terms  $A(n,m)$  for  $n = 1$  to  $n =$  total number of room surfaces and for  $m =$  (total number of room surfaces + 1):

$$A(n,m) = (-h_{cn} / \varepsilon_n) + \sum_{i=1}^N h_{ci} F_{n,i} (1 - \varepsilon_n) / \varepsilon_i \quad (\text{A8.22})$$

Terms  $A(n,m)$  for  $n = 1$  to  $n =$  total number of room surfaces and for  $m =$  (total number of room surfaces + 2):

$$A(n,m) = -R / \Sigma A \quad (\text{A8.23})$$

(b) Control sensor heat balance equations

Terms  $A(n,n)$  for  $n =$  total number of room surfaces + 1:

$$A(n,n) = F_a \quad (\text{A8.24})$$

where  $F_a$  is the fraction of the air temperature detected by the sensor. ( $F_a = 0.5$  for a sensor detecting operative temperature.)

Terms  $A(n,m)$  for  $n =$  (total number of room surfaces + 1) and for  $m = 1$  to  $m =$  total number of room surfaces:

$$A(n,m) = (1 - F_a) A_n / \Sigma A_n \quad (\text{A8.25})$$

Terms  $A(n,m)$  for  $n =$  (total number of room surfaces + 1) and for  $m =$  (total number of room surfaces + 2):

$$A(n,m) = R (1 - F_a) / (h_r \Sigma A) \quad (\text{A8.26})$$

(c) Convection heat balance

Terms  $A(n,n)$  for  $n =$  (total number of room surfaces + 2):

$$A(n,n) = (R - 1) / \Sigma A \quad (\text{A8.27})$$

Terms  $A(n,m)$  for  $n =$  (total number of room surfaces + 2) and for  $m = 1$  to  $m =$  total number of room surfaces:

$$A(n,m) = -h_{cn} A_n / \Sigma A \quad (\text{A8.28})$$

Terms  $A(n,m)$  for  $n =$  (total number of room surfaces + 2) and for  $m =$  (total number of room surfaces + 1):

$$A(n,m) = [C_v + \Sigma (A_n h_{cn})] / \Sigma A \quad (\text{A8.29})$$

Vector  $\mathbf{C}$ :

Terms  $C(n)$  for  $n = 1$  to  $n =$  the total number of room surfaces:

$$C(n) = (\theta_{on} U_n' / \varepsilon_n) - \sum_{i=1}^N [F_{n,i} U_n' \theta_{oi} (1 - \varepsilon_i) / \varepsilon_i] \quad (\text{A8.30})$$

Terms  $C(n)$  for  $n =$  (total number of room surfaces + 1):

$$C(n) = \theta_c \quad (\text{A8.31})$$

Terms  $C(n)$  for  $n =$  (total number of room surfaces + 2):

$$C(n) = \theta_{ao} C_v / \Sigma A \quad (\text{A8.32})$$

Solution vector  $\mathbf{X}$ :

Terms  $X(n)$  for  $n = 1$  to  $n =$  total number of room surfaces provide the temperatures for each surface.

Term  $X(n)$  for  $n =$  (total number of room surfaces + 1) provides the room air temperature.

Term  $X(n)$  for  $n =$  (total number of room surfaces + 2) provides the emitter output (i.e. sum of convective and radiant outputs).

The ventilation transmittance is represented by the conventional term  $C_v$ , see equation 5.34. If it is necessary to take account of air flows from a number of sources, that term in matrix  $\mathbf{A}$  should be replaced by the summation  $\Sigma (\dot{m}_a c_{p,i})$ , where the summation covers all sources  $i$ .

In vector  $\mathbf{C}$ , the term  $(\theta_{ao} C_v / \Sigma A)$  is then replaced by  $[\Sigma (\theta_i \dot{m}_a c_{p,i}) / \Sigma A]$  where  $\theta_i$  is the temperature of air from source  $i$ .

View factors are not easy to calculate and while some standard relationships are given in chapter 3 of CIBSE Guide C (2007), these will not cover many applications. Figure 5.A8.1 and the following algorithm enables view factors to be determined for rectangular rooms (ASHRAE, 1976).

## (a) Two parallel room surfaces

Radiation shape factor ( $F_{1-2}$ ) between parallel surfaces 1 and 2 separated by a distance  $G$ , see Figure 5.A8.1(a), is given by:

$$\begin{aligned} 2\pi(b_1 - a_1)(d_1 - c_1)F_{1-2} = & \{[P(b_2 - b_1) + P(a_2 - a_1)] \times [Q(c_2 - c_1) + Q(d_2 - d_1) \\ & - Q(c_2 - d_1) - Q(d_2 - c_1)]\} \\ & + \{[P(b_2 - a_1) + P(a_2 - b_1)] \times [Q(c_2 - d_1) \\ & + Q(d_2 - c_1) - Q(c_2 - c_1) - Q(d_2 - d_1)]\} \end{aligned} \quad (\text{A8.33})$$

$P$  and  $Q$  are functions; expanding equation 5.97 gives products of the form  $P(b_2 - b_1)Q(c_2 - c_1)$ , given by:

$$\begin{aligned} P(Z_1)Q(Z_2) = & Z_1 W \tan^{-1}(Z_1 / W) \\ & + Z_2 V \tan^{-1}(Z_2 / V) - (G^2 / 2) \ln[(W^2 + Z_1^2) / W_2^2] \end{aligned} \quad (\text{A8.34})$$

where  $Z_1$  and  $Z_2$  are generalised variables, e.g.  $Z_1 = (b_2 - b_1)$  and  $Z_2 = (c_2 - c_1)$ , and:

$$V^2 = G^2 + Z_{12} \quad (\text{A8.35})$$

$$W^2 = G^2 + Z_2^2 \quad (\text{A8.36})$$

## (b) Two perpendicular room surfaces

Radiation shape factor ( $F_{1-2}$ ) between perpendicular surfaces 1 and 2, see Figure 5.A8.1(a), is given by:

$$\begin{aligned} 2\pi(b_1 - a_1)(d_1 - c_1)F_{1-2} = & \{[R(b_2 - b_1) + R(a_2 - a_1)] \\ & \times [S(c_2 - c_1) + S(d_2 - d_1) - S(c_2 - d_1) - S(d_2 - c_1)]\} \\ & + \{[R(b_2 - a_1) + R(a_2 - b_1)] \times [S(c_2 - d_1) \\ & + S(d_2 - c_1) - S(c_2 - c_1) - S(d_2 - d_1)]\} \end{aligned} \quad (\text{A8.37})$$

$R$  and  $S$  are functions; expanding equation A8.37 gives products of the form  $R(b_2 - b_1)S(c_2 - c_1)$ , given by:

$$\begin{aligned} R(Z_1)S(Y_2 - Y_1) = & T Z_1 \tan^{-1}(Z_1 / T) \\ & + 1/4(Z_1^2 - T^2) \ln(T^2 + Z_1^2) \end{aligned} \quad (\text{A8.38})$$

where  $Z_1$  and  $(Y_2 - Y_1)$  are generalised variables, as above, and:

$$T^2 = Y_2^2 + Y_1^2 \quad (\text{A8.39})$$

These equations when combined with view factor algebra will satisfy the majority of needs. The relevant view factor algebra is as follows.

For conservation of energy:

$$\sum_{m=1}^M F_{n,m} = 1.0 \quad (\text{A8.40})$$

where the summation is over all surfaces comprising the enclosure.

For reciprocity:

$$A_n F_{n,m} = A_m F_{m,n} \quad (\text{A8.41})$$

If surface  $m$  is constructed from a number of sub-surfaces, e.g. windows, doors, wall, then:

$$A_n F_{n,m} = A_n F_{n,m1} + A_n F_{n,m2} + \dots \quad (\text{A8.42})$$

where surface  $m$  is made up from sub-surfaces  $m1, m2$  etc.

For cases where non-rectangular or concealed surfaces are involved or where rooms are not orthogonal, numerical techniques will be necessary for calculating view factors. These methods usually make use of contour integration (Walton, 1986) although statistically based methods have also been used (Malalasekera, 1993). The application of these methods is outside of the scope of this Guide.

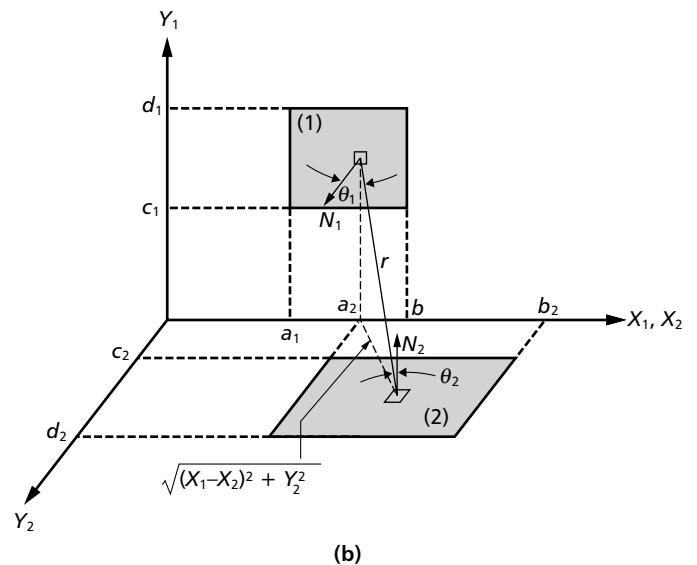
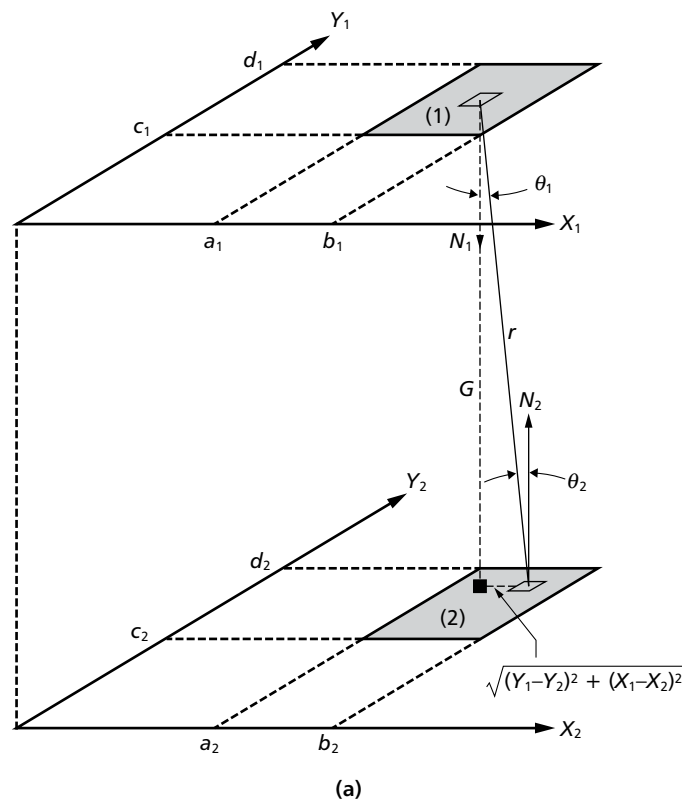


Figure 5.A8.1 View factors for radiation heat exchange; (a) between two parallel room surfaces, (b) between two perpendicular room surfaces

### 5.A8.4 Basic model

The reference model above considered the heat transfer process within a room from the view of direct surface-to-surface radiant heat flows, surface-to-air convection and surface-to-outside conduction. The surface-to-surface radiant flow is the most difficult of these processes to model. An alternative approach is to assume that just as all convective heat input must first increase the air temperature, i.e. enters the 'air temperature node' so all radiant heat enters at the 'radiant temperature node' ( $\theta^*$ ). Heat then flows into each room surface by means of a heat transfer coefficient that is adjusted to take account of the multiple reflections of radiation between surfaces. Davies (1990) has shown that it is possible to make a very close approximation to the exact equivalent of the radiosity matrix used in the reference model for a six-sided enclosure.

In such a case the radiant heat transfer coefficient is equal to the product ( $E_n^* h_r$ ) where:

$$E_n^* = \varepsilon_n / (1 - \varepsilon_n + \beta_n \varepsilon_n) \quad (\text{A8.43})$$

where  $\beta_n$  is given by the regression equation:

$$\beta_n = 1 - f_n [1 + 3.53 (f_n - 0.5) - 5.04 (f_n^2 - 0.25)] \quad (\text{A8.44})$$

where:

$$f_n = A_n / \Sigma A \quad (\text{A8.45})$$

The standard error for the regression is 0.0068 and the exact value of  $\beta_n$  for a cube is 5/6.

While the terms  $\beta_n$  are specific to a six-sided enclosure, they can often be used in most design applications. Introducing this close approximation to the radiant exchange process, greatly simplifies matrix  $A$  at the minor expense of the introduction of a new temperature  $\theta^*$ , as follows:

$$\sum_{n=1}^N h_{cn} A_n + \dot{m}_a c_p \theta_{ai} - \sum_{n=1}^N h_{cn} A_n \theta_{sn} = \Phi_{con} + \theta_{ao} \dot{m}_a c_p \quad (\text{A8.46})$$

$$\sum_{n=1}^N h_r^* E_n^* A_n \theta^* - \sum_{n=1}^N h_r^* E_n^* A_n \theta_{sn} = \Phi_n + \Phi_{rad} \quad (\text{A8.47})$$

where  $\Phi_{con}$  is the convective output from an emitter (W) and  $\Phi_{rad}$  is the radiant output (W).

For each surface  $n$ :

$$\begin{aligned} -A_n h_{cn} \theta_{ai} - A_n E_n^* h_r \theta^* + (h_{cn} + E_n^* h_r + U_n') A_n \theta_{sn} \\ = \Phi_n + \theta_{on} A_n U_n' \end{aligned} \quad (\text{A8.48})$$

where  $\Phi_n$  is a heat input to surface  $n$  (W), e.g. the absorbed solar radiation incident upon the surface. For the purposes of a heat loss model, all  $\Phi_n$  are set to zero.

The basic model is given by the equation set represented by the matrix equation:

$$A^* X^* = C^* \quad (\text{A8.49})$$

where  $A^*$ ,  $X^*$  and  $C^*$  are matrices, as defined in the following boxes.

Matrix  $A^*$ :

(a) Surface heat balance

Terms  $A^*(n,n)$  for  $n = 1$  to  $n =$  total number of room surfaces:

$$A^*(n,n) = (h_{cn} + U_n' + E_n^* h_r) \quad (\text{A8.50})$$

Terms  $A^*(n,m)$  where  $n \neq m$ , for  $n = 1$  to  $n =$  total number of room surfaces and for  $m = 1$  to  $m =$  total number of room surfaces:

$$A^*(n,m) = 0 \quad (\text{A8.51})$$

Terms  $A^*(n,m)$  for  $n = 1$  to  $n =$  total number of room surfaces and for  $m =$  (total number of room surfaces + 1):

$$A^*(n,m) = -h_{cn} \quad (\text{A8.52})$$

Terms  $A^*(n,m)$  for  $n = 1$  to  $n =$  total number of room surfaces and for  $m =$  (total number of room surfaces + 2):

$$A^*(n,m) = 0 \quad (\text{A8.53})$$

Terms  $A^*(n,m)$  for  $n = 1$  to  $n =$  total number of room surfaces and for  $m =$  (total number of room surfaces + 3):

$$A^*(n,m) = -E_n^* h_r \quad (\text{A8.54})$$

(b) Control sensor heat balance

Terms  $A^*(n,n)$  for  $n =$  (total number of room surfaces + 1):

$$A^*(n,n) = F_a \quad (\text{A8.55})$$

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 1) and for  $m = 1$  to  $m =$  total number of room surfaces:

$$A^*(n,m) = \varepsilon_m (1 - F_a) A_m / \Sigma (A \varepsilon) \quad (\text{A8.56})$$

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 1) and for  $m =$  (total number of room surfaces + 2):

$$A^*(n,m) = R (1 - F_a) / (h_r \Sigma A) \quad (\text{A8.57})$$

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 1) and for  $m =$  (total number of room surfaces + 3):

$$A^*(n,m) = 0 \quad (\text{A8.58})$$

(c) Convection heat balance

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 2) and for  $m = 1$  to  $m =$  total number of room surfaces:

$$A^*(n,m) = -h_{cn} A_m / \Sigma (A \varepsilon) \quad (\text{A8.59})$$

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 2) and for  $m =$  (total number of room surfaces + 1):

$$A^*(n,m) = [C_v + \sum (A_m h_{cm})] / \sum (A \varepsilon) \quad (A8.60)$$

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 2) and for  $m =$  (total number of room surfaces + 2):

$$A^*(n,m) = (R - 1) / \sum A \quad (A8.61)$$

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 2) and for  $m =$  (total number of room surfaces + 3):

$$A^*(n,m) = 0 \quad (A8.62)$$

(d) Radiant heat balance

Terms  $A^*(n,n)$  for  $n =$  (total number of room surfaces + 3):

$$A^*(n,n) = \sum E_n^* h_r A_n / \sum A \quad (A8.63)$$

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 3) and for  $m = 1$  to  $m =$  total number of room surfaces:

$$A^*(n,m) = -E_m^* h_r A_m / \sum A \quad (A8.64)$$

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 3) and for  $m =$  (total number of room surfaces + 1):

$$A^*(n,m) = 0 \quad (A8.65)$$

Terms  $A^*(n,m)$  for  $n =$  (total number of room surfaces + 3) and for  $m =$  (total number of room surfaces + 2):

$$A^*(n,m) = -R / \sum A \quad (A8.66)$$

Vector  $C^*$ :

Terms  $C^*(n)$  for  $n = 1$  to  $n =$  the total number of room surfaces:

$$C^*(n) = \theta_{on} U_n' \quad (A8.67)$$

Terms  $C^*(n)$  for  $n =$  (total number of room surfaces + 1):

$$C^*(n) = \theta_c \quad (A8.68)$$

Terms  $C^*(n)$  for  $n =$  (total number of room surfaces + 2):

$$C^*(n) = \theta_{ao} C_v / \sum A \quad (A8.69)$$

Terms  $C^*(n)$  for  $n =$  total number of room surfaces + 3:

$$C^*(n) = 0 \quad (A8.70)$$

Solution vector  $X^*$ :

Terms  $X^*(n)$  for  $n = 1$  to  $n =$  total number of room surfaces provide the temperature of each surface.

Term  $X^*(n)$  for  $n =$  (total number of room surfaces + 1) provides the room air temperature.

Term  $X^*(n)$  for  $n =$  (total number of room surfaces + 2) provides the total heat input.

Term  $X^*(n)$  for  $n =$  (total number of room surfaces + 3) provides the radiant-star temperature ( $\theta^*$ ), which is not the radiant temperature.

The ventilation transmittance is represented by the conventional term  $C_v$ , see equation 5.34. If it is necessary to take account of air flows from a number of sources, that term in matrix  $A$  should be replaced by the summation  $\sum (\dot{m}_a c_{p,i})$  where the summation covers all sources  $i$ .

In vector  $C$ , the term  $(\theta_{ao} C_v / \sum A)$  is then replaced by  $[\sum (\theta_i \dot{m}_a c_{p,i}) / \sum A]$  where  $\theta_i$  is the temperature of air from source  $i$ .

### 5.A8.5 Simple model

If the radiant exchange between surfaces can be treated separately, the surface heat balance equations are decoupled and the need for matrix manipulation is removed. This leads to a manual calculation procedure.

One means of achieving this approximation is to assume that, with the exception of the subject surface, all surface temperatures are known. In this case, the heat balance on the subject surface is described by the surface heat balance equations given for the basic model, see equation A8.48. Hence:

$$\theta_s (h_c + U' + E^* h_r) - h_c \theta_{ai} - E^* h_r \theta^* = \theta_o U' \quad (A8.71)$$

Rearranging equation A8.71 gives the fabric heat loss:

$$U' (\theta_s - \theta_o) = h_c (\theta_{ai} - \theta_s) + h_r E^* (\theta^* - \theta_s) \quad (A8.72)$$

It then remains to determine a value for  $E^*$ . A simple method should use parameters that are independent of the shape of the enclosure. The simplest assumption is that the subject surface has an area equivalent to one sixth of that of the enclosure of which it forms a part. Therefore, from equation A8.43, with  $f_n = 1/6$  (see equation A8.49) and  $\beta_n = 5/6$  (see equation A8.44):

$$E^* = \frac{\varepsilon}{(1 - \varepsilon + 5/6 \varepsilon)} \quad (A8.73)$$

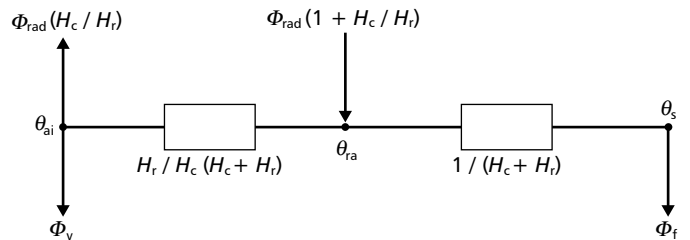


Figure 5.A2.2 Simplified heat flow network

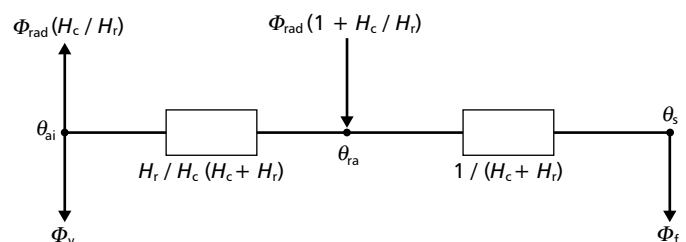


Figure 5.A2.3 Equivalent heat flow network

For  $\varepsilon = 1$ ,  $E^* = 6/5 \varepsilon$ :

$$\Phi_f = h_c (\theta_{ai} - \theta_s) + 6/5 \varepsilon h_r (\theta^* - \theta_s) \quad (\text{A8.74})$$

Equation A8.74 may be summed for all surfaces to give the total fabric loss, that is:

$$\Phi_f = h_c \Sigma A (\theta_{ai} - \theta_m) + 6/5 \varepsilon h_r \Sigma A (\theta^* - \theta_m) \quad (\text{A8.75})$$

where it is assumed that  $h_c$  and  $h_r$  are constants and that:

$$\theta_m = \Sigma A \theta_s / \Sigma A \quad (\text{A8.76})$$

The heat input to a space comprises a radiant and convective component. From equation A8.47, the radiant component is:

$$\Phi_{rad} = 6/5 \varepsilon h_r \Sigma A (\theta^* - \theta_m) \quad (\text{A8.77})$$

The convective components associated with the fabric heat loss is:

$$\Phi_{con} = h_c \Sigma A (\theta_{ai} - \theta_m) \quad (\text{A8.78})$$

Equations A8.79, A8.77 and A8.78 can be expressed in analogue form by the network shown in Figure 5.A8.2, where:

$$H_c = h_c \Sigma A \quad (\text{A8.79})$$

and:

$$H_r = 6/5 \varepsilon h_r \Sigma A \quad (\text{A8.80})$$

Figure 5.A8.2 shows a radiant input  $\Phi_{rad}$ , acting at the radiant star node  $\theta^*$ , being lost by conduction  $\Phi_f$  from  $\theta_s$  and by ventilation  $\Phi_v$  from  $\theta_{ai}$ . This network may be transformed exactly into that shown in Figure 5.A8.3 where the rad-air node  $\theta_{ra}$  is located on the convective transmittance  $H_c$ , dividing it into two components:  $X = H_c (H_c + H_r) / H_r$  and  $Y = (H_c + H_r)$ . An augmented flow,  $\Phi_{rad} (1 + H_c / H_r)$  acts at  $\theta_{ra}$  and the excess,  $\Phi_{rad} (H_c / H_r)$  is withdrawn from  $\theta_{ai}$ . Components  $X$  and  $Y$  can be considered, in effect, in parallel (Davies, 1990). The physically significant quantities, i.e. the observable temperatures  $\theta_s$  and  $\theta_{ai}$ , and the heat flows from them,  $\Phi_f$  and  $\Phi_v$ , can be considered the same in both cases.

There is a further transmittance,  $[(H_c + H_r) H_c / H_r]$ , between  $\theta_{ra}$  and  $\theta_{ai}$ . The rad-air temperature,  $\theta_{ra}$ , is related to the two generating temperatures by the following equation:

$$\theta_{ra} = \frac{H_c \theta_{ai}}{H_c + H_r} + \frac{H_r \theta^*}{H_c + H_r} \quad (\text{A8.81})$$

If the mean surface temperature,  $\theta_m$ , is taken as an approximation for  $\theta^*$ , then  $\theta_{ra} \approx \theta_{ei}$ . Hence:

$$\theta_{ei} = \frac{H_c \theta_{ai}}{H_c + H_r} + \frac{H_r \theta_m}{H_c + H_r} \quad (\text{A8.82})$$

It is appropriate to standardise the heat transfer coefficients as follows:

$$h_c = 3.0 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1} \text{ (an average figure)}$$

$$h_r = 5.7 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1} \text{ (for temperatures } \approx 20^\circ \text{C)}$$

$$H_r / \Sigma A = 6.0 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1} \text{ (for } \varepsilon = 0.9)$$

$$H_c / \Sigma A = 3.0 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$$

Also:

$$H_a / \Sigma A = (H_r + H_c) H_c / H_r = 4.5 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$$

So:

$$h_a = 4.5 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$$

Therefore, it follows from equation A8.82 that:

$$\theta_{ei} = 1/3 \theta_{ai} + 2/3 \theta_m \quad (\text{A8.83})$$

That is, the effective radiant heat input is 1.5 times the actual input, with the excess (50%) of radiant input subtracted from the convective component of the heat input. A further implication is that a heat source that is effectively directly linked to the environmental temperature has the characteristics of  $2/3$  radiation and  $1/3$  convection.

It is accepted that a number of approximations are embodied in this relationship. However, empirical testing over a number of years has not revealed any serious deficiencies in practice and, as such, it is therefore accepted as the basis of the CIBSE simple heat loss model, which is developed as follows.

The heat loss due to the fabric is defined as:

$$\Phi_f = \sum_{n=1}^N A_n U_n (\theta_{ei} - \theta_{on}) \quad (\text{A8.84})$$

Where the fabric term contains heat loss through internal partitions, a modified  $U$ -value should be used:

$$U_p = \frac{U (\theta_c - \theta_c')}{(\theta_c - \theta_{ao})} \quad (\text{A8.85})$$

This correction is based on the internal design operative temperature ( $\theta_c$ ) and therefore is not exact. However, the operative is usually very close to the heat loss temperature ( $\theta_{ei}$ ) which means that any error is small in what is already a second order correction. This approximation makes it unnecessary to determine the value of the environmental temperature in adjacent spaces.

The heat loss due to infiltration and/or ventilation by outdoor air is:

$$\Phi_v = \frac{c_p \rho N_v V}{3600} (\theta_{ai} - \theta_{ao}) \quad (\text{A8.86})$$

For practical purposes  $(c_p \rho / 3600) = 1/3$ , therefore  $C_v = N_v V / 3$ .



Hence:

$$\Phi_v = C_v (\theta_{ai} - \theta_{ao}) \quad (\text{A8.87})$$

Ventilation rates must include infiltration, natural ventilation due to open windows and, where appropriate, mechanical ventilation. Guidance on ventilation requirements and design allowances for infiltration are given in chapter 1 and chapter 4 of this Guide, respectively.

The total heat loss is the sum of the fabric and infiltration losses:

$$\Phi_t = \sum_{n=1}^N A_n U_n (\theta_{ei} - \theta_{on}) + C_v (\theta_{ai} - \theta_{ao}) \quad (\text{A8.88})$$

For winter heating design conditions it is conventional to assume that the outside heat transfer temperature ( $\theta_{on}$ ) equals the outside air temperature ( $\theta_{ao}$ ), therefore:

$$\Phi_t = \sum_{n=1}^N A_n U_n (\theta_{ei} - \theta_{ao}) + C_v (\theta_{ai} - \theta_{ao}) \quad (\text{A8.89})$$

In order to relate the heat loss to the design operative temperature, it is necessary to eliminate  $\theta_{ai}$  and  $\theta_{ei}$ . This is achieved by introducing factors  $F_{1cu}$  and  $F_{2cu}$ , as follows (see Appendix 5.A2, equations A2.17 and A2.18:

$$F_{1cu} = \frac{3.0 (C_v + 6 \Sigma A)}{\Sigma (A U) + 18 \Sigma A + 1.5 R [3 C_v - \Sigma (A U)]} \quad (\text{A8.90})$$

$$F_{2cu} = \frac{\Sigma (A U) + 18 \Sigma A}{\Sigma (A U) + 18 \Sigma A + 1.5 R [3 C_v - \Sigma (A U)]} \quad (\text{A8.91})$$

Therefore the simple model is:

$$\Phi_t = (F_{1cu} \sum_{n=1}^N A_n U_n + F_{2cu} C_v) (\theta_c - \theta_{ao}) \quad (\text{A8.92})$$

and the corresponding air temperature is calculated using the following equation (see Appendix 5.A2, equation A2.20):

$$\bar{\theta}_{ai} = \frac{\bar{\Phi}_t (1 - 1.5 R) + C_v \bar{\theta}_{ao} + 6.0 \Sigma A \bar{\theta}_c}{C_v + 6.0 \Sigma A} \quad (\text{A8.93})$$

## References for Appendix 5.A8

ASHRAE (1976) *Energy calculation procedures to determine heating and cooling loads for computer analysis* (Atlanta GA: American Society of Heating Refrigerating and Air-conditioning Engineers)

CIBSE (2001) *Heat transfer* ch. 3 in CIBSE Guide C: *Reference data* (London: Chartered Institution of Building Services Engineers)

Davies MG (1990) 'An idealised model for room radiant exchange' *Building and Environment* 25(4) 375–378

Gagneau S, Nataf JM and Wurtz E (1997) 'An illustration of automatic generation of zonal models' *Proc. Int. Building Performance Simulation Association Conf., Prague, Czech Republic* 2 437–444

Malalasekera WMG and James EH (1993) 'Thermal radiation in a room: numerical evaluation' *Building Serv. Eng. Res. Technol.* 14(4) 159–168

Walton GN (1986) *Algorithms for calculating radiation view factors between plane convex polygons with obstructions* NBSIR 86-3463 (Washington DC: US Department of Commerce)

## Appendix 5.A9: Comparison of thermal steady-state models

Although the methods described in this appendix are presented as steady state methods they can be, and are, the basis of a room model in a transient calculation, Of these methods the only one suited to hand calculation is the ‘simple model’. The ‘basic model’ can easily be implemented within a spreadsheet but the ‘reference model’ is best used within a software package. The CIBSE recognises that, although many users will not have the luxury of choice of method, users should understand the limitations of the methodology employed within any software application.

The significant difference between the methods lies in the way heat transfer by longwave radiation is treated with the consequence that, assuming a uniform distribution of air temperature within the space, simplification results in errors in the calculation of surface temperature. In the case of the simple model all surfaces with the same *U*-value and adjacent to the same bounding temperature will be predicted to have the same internal surface temperature; for the basic model surfaces with identical areas and *U*-values will have identical surface temperatures. Thus these models may not be appropriate for studies associated with prediction of surface mould growth or condensation\*. Predictions of heating and cooling load will also differ. However, in most cases differences will be a few percent and virtually zero with well insulated buildings. This is demonstrated in Example 5.A9.1, below, where the highlighted surfaces are

of identical areas. The calculation is for a convective heating system.

### Example 5.A9.1: Enclosure with multiple surfaces; varying *U*-values, uniform emissivities

See Figure 5.A9.1 and Tables 5.A9.1 and 5.A9.2. In this example, the calculation methods are applied to multiple surfaces. It is intended to assist in checking computer programs to determine the accuracy with which surface temperatures are calculated.

Operative temperature: 21 °C  
 Outside air temperature: -1 °C  
 Infiltration rate: 1.0 h<sup>-1</sup>

\* In many cases, these three-dimensional heat transfer models will be required to understand the implications of thermal bridges. However, the reference model can provide a good indication of the potential for condensation

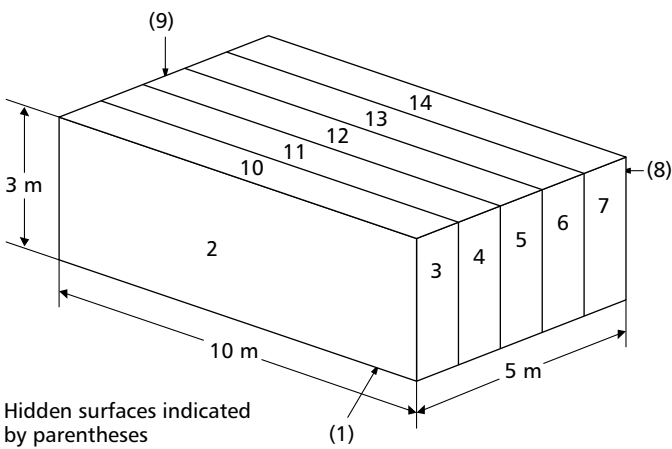


Figure 5.A9.1 Example 5.A9.1: Geometry for enclosure with multiple surfaces

Table 5.A9.1 Example 5.A9.1: surface data

Surface number	Area / m <sup>2</sup>	<i>U</i> -value / W·m <sup>-2</sup> ·K <sup>-1</sup>	Emissivity of surface, $\epsilon_n$	Convective heat transfer coefficient, $h_c$	Inside surface resistance, $R_{si}$ / m <sup>2</sup> ·K·W <sup>-1</sup>	Temperature on outer side of surface / °C
1	50.0	1.0	0.8	1.5	0.14	-1.0
2	30.0	5.6	0.8	3.0	0.12	-1.0
3	3.0	1.0	0.8	3.0	0.12	-1.0
4	3.0	1.0	0.8	3.0	0.12	-1.0
5	3.0	1.0	0.8	3.0	0.12	-1.0
6	3.0	1.0	0.8	3.0	0.12	-1.0
7	3.0	1.0	0.8	3.0	0.12	-1.0
8	30.0	1.0	0.8	3.0	0.12	-1.0
9	15.0	1.0	0.8	3.0	0.12	-1.0
10	10.0	1.0	0.8	4.3	0.10	-1.0
11	10.0	1.0	0.8	4.3	0.10	-1.0
12	10.0	1.0	0.8	4.3	0.10	-1.0
13	10.0	1.0	0.8	4.3	0.10	-1.0
14	10.0	1.0	0.8	4.3	0.10	-1.0

Figure 5.A9.2 Example 5.A9.1: Comparison of results of example calculation using the reference, basic and simple models

Property	Calculation method		
	Reference	Basic	Simple
Air temperature (°C)	27.03	27.05	26.78
Mean radiant temp. (°C)	14.97	14.95	15.22
Surface temp. (°C):			
— surface no. 1	15.07	14.82	16.2
— surface no. 2	5.74	5.72	5.81
— surface no. 3	16.06	16.98	16.71
— surface no. 4	16.71	16.98	16.71
— surface no. 5	17.07	16.98	16.71
— surface no. 6	17.3	16.98	16.71
— surface no. 7	17.46	16.98	16.71
— surface no. 8	16.94	16.85	16.71
— surface no. 9	16.92	16.93	16.71
— surface no. 10	17.26	18.29	17.02
— surface no. 11	17.81	18.29	17.02
— surface no. 13	18.18	18.29	17.02
— surface no. 13	18.42	18.29	17.02
— surface no. 14	18.61	18.29	17.02
Heat input (W)	7962	7949	7973
Fabric loss (W)	6560	6547	6585
Air loss (W)	1402	1402	1389

## Appendix 5.A10: Algorithm for the calculation of cooling loads by means of the admittance method

### 5.A10.1 Introduction

This appendix describes the way room cooling loads are calculated using the CIBSE admittance method. The solar position and transmission algorithms are those used to produce the cooling load tables in the 1999 and earlier editions of CIBSE Guide A. Other calculations follow the equations presented in Appendix 5.A2 which, although different in appearance, are identical to those in editions of Guide A preceding the 1999 edition.

The solar cooling load tables are based upon a particular space and rules related to the use of blinds. Details are given in section 5.A10.17.

An example of the implementation of this method is given by White et al. and currently a software tool can be downloaded via <http://www.arup.com> and searching for PDA. Note that the CIBSE does not endorse software. The tool should simply be seen as an example of the implementation of the admittance method.

### 5.A10.2 Input data

The input data required are:

- Latitude of the building: the calculations here are carried out for Local Apparent Time (solar time) and so longitude is not required. Information on how to correct to clock time is given in the annex to chapter 2 of this Guide, which may be downloaded from the CIBSE website\*.
- Internal design temperature: operative temperature, see chapter 1, Table 1.5.
- Hourly dry bulb temperatures: design values can be found in chapter 2, Tables 2.14(a) to 2.14(n), which may be downloaded from the CIBSE website\*.
- Hourly values of direct and diffuse solar radiation: design values can be found in chapter 2, Tables 2.13(a) to 2.13(n), which may be downloaded from the CIBSE website\*.
- Dimensions of the space.
- Material properties: these are the dimensions and thermal properties of the fabric elements bounding the space. For glazing the data must be sufficient to determine the transmission, absorption and reflection for each pane of glass as a function of the solar angle of incidence.
- Internal heat gains: the hourly profile of use and the radiant/convective split are required.
- Infiltration rate/ventilation rate: this is for outside air only and it is assumed here to be constant throughout the day. The calculation of cooling loads when the ventilation rate varies is described in section 5.A2.7.

- Boundary conditions for internal surfaces: this algorithm assumes that internal surfaces are adiabatic. If the temperature in adjacent spaces is known then they can be treated in the same way as external spaces. If not an iterative procedure is required.
- Time plant is switched on and off.

### 5.A10.3 Overview

The basic process is as follows, for each hour of the day. Note that if measured climatic data are used the solar radiation at any given hour is, usually, the average over the preceding hour and so calculations should be made on the half hour. Measured temperatures are usually reported on the hour and so interpolation may be required in order to obtain the half hour value.

- (1) Calculate the  $U$ -value, thermal admittance, decrement and surface factor for all fabric elements (see chapter 3).
- (2) Calculate the factors required by the method.

The following preliminary calculations are carried out for each hour of the day:

- (3) Calculate the position of the sun.
- (4) Generate the direct and diffuse radiation normal to the sun (or obtain from tabulated or measured data).
- (5) Obtain appropriate hourly dry bulb temperatures.
- (6) Calculate the sol-air temperature for all external surfaces.
- (7) Calculate the radiation transmitted through and absorbed within the glazing. If necessary allowing for external shading devices and the raising or lowering of blinds. It is assumed here that blinds are lowered because of external conditions (level of solar radiation) and not internal space temperature. Iteration will be necessary if internal temperature control is required.

This completes the preliminary calculations.

- (8) The following loads at the environmental and air node are needed (see 5.A2.3 and 5.A2.4). Note that in the case of the solar cooling load tables only the solar and infiltration loads are required.
  - solar
  - infiltration/ventilation
  - fabric
  - internal gains.
- (9) Sum the gains and determine the cooling load for 24-hour plant operation.
- (10) Apply correction for intermittent plant operation.

The method of calculation is given in the following sections.

\* <http://www.cibse.org/Guide-A/pdfs>

### 5.A10.4 Correction factors

The calculation requires the following input data for each surface:

VOL	Room volume (m <sup>3</sup> )
AWALL	Opaque area (m <sup>2</sup> )
AGLASS	Glazed area (m <sup>2</sup> )
U	Thermal transmittance ( <i>U</i> -value)
Y	Thermal admittance ( <i>Y</i> -value)
YL	Time lead associated with thermal admittance
D	Decrement factor
DL	Time delay associated with the decrement factor
SF	The surface factor
SFD	The time delay associated with the surface factor
AIRCH	The air change rate
PLNTON	Time plant switched on
PLNTOFF	Time plant switched off

The following summations are necessary, the derived *U*-values etc. are those appropriate to an individual surface.

SIGA	Sum of all AWALL and AGLASS
SIGAU	Sum of all AWALL* <i>U</i> and AGLASS* <i>U</i>
SIGAY	Sum of all AWALL* <i>Y</i> and AGLASS* <i>Y</i>
SIGAYL	Sum of all AWALL* <i>YL</i> and AGLASS* <i>YL</i>
SIGASF	Sum of all AWALL* <i>SF</i> and AGLASS* <i>SF</i>
SIGASFD	Sum of all AWALL* <i>SFD</i> and AGLASS* <i>SFD</i>

Ventilation conductance:

$$CV = AIRCH * VOL / 3$$

Response factor:

$$RFACT = (SIGAY + CV) / (SIGAU + CV)$$

Non-dimensional factors:

$$FU = 18 * SIGA / (18 * SIGA + SIGAU)$$

$$FY = 18 * SIGA / (18 * SIGA + SIGAY)$$

$$FV = 6 * SIGA / (6 * SIGA + CV)$$

$$F1A = 4.5 * SIGA / ((1 - 1.5R) * SIGAU + 4.5 * SIGA)$$

$$F2A = (SIGAU + 4.5 * SIGA) / ((1 - 1.5R) * SIGAU + 4.5 * SIGA)$$

$$F1AY = 4.5 * SIGA / ((1 - 1.5R) * SIGAY + 4.5 * SIGA)$$

$$F2AY = (SIGAY + 4.5 * SIGA) / ((1 - 1.5R) * SIGAY + 4.5 * SIGA)$$

$$F1C = 3.0 * (CV + 6 * SIGA) / ((SIGAU + 18.0 * SIGA + 1.5 * R * (3.0 * CV - SIGAU))$$

$$F2C = (SIGAU + 18.0 * SIGA) / (SIGAU + 18.0 * SIGA + 1.5 * R * (3.0 * CV - SIGAU))$$

$$F1CY = 3.0 * (CV + 6.0 * SIGA) / (SIGAY + 18.0 * SIGA + 1.5 * R * (3.0 * CV - SIGAY))$$

$$F2CY = (SIGAY + 18.0 * SIGA) / (SIGAY + 18.0 * SIGA + 1.5 * R * (3.0 * CV - SIGAY))$$

Factor for correction for intermittent operation:

$$PRUN = PLTOFF - PLTON + 1$$

$$DOUTPT = (FY * SIGAY - FU * SIGAU) / (24 - PRUN) * FU * SIGAU + PRUN * FY * SIGAY + 24 * CV * FV$$

Admittance and the associated factors are vector quantities and so all delays and leads should be handled separately. This simple method assumes that the overall response to solar radiation can be represented by a mean value for the surface factor as follows.

Mean surface factor:

$$SFBAR = SIGASF / SIGA$$

Mean surface factor delay:

$$SFDEL = SIGASFD / SIGA$$

The delay is rounded to the nearest hour, but if zero set to 1 hour.

### 5.A10.5 Calculation of solar position.

The calculation requires the following input data:

RLAT	Latitude (radians)
NUMDAY	Day of year (January 1st = 1, December 31st = 365)
HOUR	Sun time (sun will be overhead at 12.00)

The calculated data are:

DECANG	Declination angle (radians)
SUNALT	Solar altitude (radians)
SUNAZI	Solar azimuth (radians)
SUNRIS	Time sun rises (decimal hours)
SUNSET	Time sun sets (decimal hours)

#### 5.A10.5.1 Declination angle

This the latitude at which the sun is overhead at solar noon.

$$SINDEC = (0.398 * \text{Sine}(0.01721 * \text{NUMDAY} + 0.03347 * \text{Sine}(0.01721 * \text{NUMDAY}) - 1.4096))$$

$$DECANG = \text{Arcsine}(SINDEC)$$

$$COSDEC = \text{Cosine}(DECANG)$$

$$TANDEC = \text{Tangent}(DECANG)$$

#### 5.A10.5.2 Solar altitude and azimuth

$$COSLAT = \text{Cosine}(RLAT)$$

$$SINLAT = \text{Sine}(RLAT)$$

$$TANLAT = \text{Tangent}(RLAT)$$

Check if TANLAT is not equal to zero then:

$$\text{TANRAT} = \text{TANDEC} / \text{TANLAT}$$

Otherwise TANRAT is equal to a large number (10E32) and given the sign of TANDEC.

Hour angle:

HANG = Absolute value (sign ignored so taken as positive) of  $((\pi * 15 / 180) * (12 - \text{HOUR}))$

COSHAG = Cosine (HANG)

SINALT =  $\text{COSLAT} * \text{COSDEC} * \text{COSHAG} + \text{SINLAT} * \text{SINDEC}$

SUNALT = Arcsine (SINALT)

COSALT = Cosine (SUNALT)

TANALT = Tangent (SUNALT)

If the solar altitude (SUNALT) is negative the sun is below the horizon, otherwise it is necessary to carry out some checks.

$$\text{TV0} = \text{COSDEC} * \text{Sine}(\text{HANG}) / \text{COSALT}$$

If TV0 is greater than 1 then it is set to 1, if it is less than -1 then it is set to -1.

C = Arcsine (TV0)

TV1 = COSHAG

TV2 = TANRAT

*Northern hemisphere*

If the sine of the latitude (SINLAT) is greater than zero (Northern hemisphere) then if the hour is before 12 the following conditional checks are necessary.

Morning:

If TV1 is greater than TV2 then  $\text{SUNAZI} = \pi - C$

If TV1 is equal to TV2 then  $\text{SUNAZI} = \pi / 2$

If TV1 is less than TV2 then  $\text{SUNAZI} = C$

*Southern hemisphere*

Switch values:

TV3 = TV1

TV1 = TV2

TV2 = TV3

The checks are now made for the afternoon:

If TV1 is greater than TV2 then  $\text{SUNAZI} = \pi + C$

If TV1 is equal to TV2 then  $\text{SUNAZI} = 1.5 * \pi$

If TV1 is less than TV2 then  $\text{SUNAZI} = 2\pi - C$

### 5.A10.5.3 Sunrise and sunset times

$$\text{COSANG} = \text{TANDEC} * \text{TANLAT}$$

*Normal situation*

Some checks:

TV4 = the absolute value of (COSANG-1)

If TV4 is negative then the time of sunrise =  $12 - \text{Arccosine}(\text{COSANG}) / \pi$ ; the time of sunset =  $24 - \text{the time of sunrise}$ .

*Other cases*

If COSANG is less than unity the sun never rises.

If COSANG is equal to or greater than unity the sun never sets.

## 5.A10.6 Calculation of the solar radiation incident upon a surface and the angle of incidence

This calculation requires the following input data where it is assumed that any corrections for sky clarity and altitude have been applied if theoretical solar data is used.

SUNAZI	Solar azimuth (radians)
SUNALT	Solar altitude (radians)
ORIEN	Surface orientation (radians, North 0 or $2\pi$ )
SLOPE	Angle of surface to horizontal (radians, flat roof 0, vertical wall $\pi/2$ )
DIRAD	Direct radiation normal to the sun ( $\text{W} \cdot \text{m}^{-2}$ )
DIFRAD	Diffuse radiation on the horizontal ( $\text{W} \cdot \text{m}^{-2}$ )
GREF	Solar albedo (Ground reflectance)
Calculated values:	
DIRECT	Direct radiation incident upon an exposed surface ( $\text{W} \cdot \text{m}^{-2}$ )
SKYDIF	Sky diffuse radiation incident on a surface ( $\text{W} \cdot \text{m}^{-2}$ )
GRDREF	Ground reflected radiation incident on a surface ( $\text{W} \cdot \text{m}^{-2}$ )
ANGINC	Solar angle of incidence (radians)

### 5.A10.6.1 Solar angle of incidence

Default this to  $\pi/2$  (i.e. the sun's rays are parallel to the surface).

The solar azimuth relative to the surface is:

$$\text{WALSUN} = \text{SUNAZI} - \text{ORIEN}$$

$$\text{COSSLO} = \text{Cosine (SLOPE)}$$

$$\text{SINSLO} = \text{Sine (SLOPE)}$$

$$\text{COSINC} = \text{Cosine (SUNALT)} * \text{SINSLO} * \text{Cosine (WALSUN)} + \text{Sine (SUNALT)} * \text{COSSLO}$$

If COSINC is positive the solar angle of incidence (ANGINC) is equal to Arccosine (COSINC).

### 5.A10.6.2 Incident radiation

Direct radiation incident on surface, first check if surface is facing the sun, for this the angle of incidence must be less than 90 degrees; that is the cosine of the angle of incidence (COSINC) is greater than zero. The intensity of the direct radiation (beam) is:

$$\text{DIRECT} = \text{DIRAD} * \text{COSINC}$$

The diffuse radiation falling on the surface depends on the orientation of the surface. The simple correction given here was used in the original calculation of the cooling load tables.

$$\text{AZCOR} = 0.9 - 0.1 * \text{Cosine (ORIEN)}$$

$$\text{AZCOR} = 1. - \text{SINSLO} * (1. - \text{AZCOR})$$

$$\text{SKYDIF} = \text{DIFRAD} * \text{AZCOR} * (1. + \text{COSSLO}) / 2.0$$

$$\text{GRDREF} = \text{GREF} * ((\text{BASRAD} * \text{Sine (SUNALT)} + \text{DIFBAS}) / 2.) * (1. - \text{COSSLO})$$

### 5.A10.7 Dry bulb temperature

Where measured values are used combined with measured solar radiation data, it is necessary to ensure consistent timing. For example, measured solar data are usually the average over the preceding hour and referenced to local apparent time (LAT) (although in the latest CIBSE hourly weather data sets solar data is referenced to GMT). Dry bulb data are usually with reference to the local time zone time.

In this case calculations should be made on the half hour. In theory, the dry-bulb time should be corrected to LAT using the 'equation of time' (see the annex to chapter 2 of this Guide, which may be downloaded from the CIBSE website\*). Bearing in mind the approximations involved in the admittance method, this is considered unnecessary for use of the method in the UK. The same may not be true where time zones span greater distances. It is therefore considered sufficient to use the average of the dry-bulb over the hour.

It is important to be consistent in timing and the convention of 'hour 1' being representative of the period midnight to 01:00 is recommended.

### 5.A10.8 Sol-air temperature

The following input data are required:

$\text{TDRY}$	External dry bulb temperature
---------------	-------------------------------

$\text{HSO}$	External surface heat transfer coefficient (see chapter 3, section 3.3.9)
--------------	---

$\text{ALPHA}$	External surface absorption coefficient
----------------	---

$\text{EMISS}$	External surface emissivity
----------------	-----------------------------

$\text{SLOPE}$	Angle of surface to horizontal (radians: flat roof 0, vertical wall $\pi/2$ )
----------------	---

$\text{RAD}$	Incident solar radiation = sum of direct, ground reflected and sky diffuse solar radiation, as appropriate.
--------------	---

$\text{RRLM}$	Longwave radiation loss; standard value for an emissivity of 1 is $100 \text{ W} \cdot \text{m}^{-2}$ .
---------------	---

Calculated value:

$\text{TSOL}$	Sol-air temperature ( $^{\circ}\text{C}$ )
---------------	--

The following allows for a reduction in longwave loss dependant on the angle between the surface and the sky. For the surface to see the sky the slope must be less than  $\pi$ . In which case the reduction factor is zero ( $\text{COR}=0$ ), otherwise:

$$\text{Let } X = \text{SURANG} / \pi$$

The correction factor is:

$$\text{COR} = 1. - X * (2. - X)$$

The longwave loss is then:

$$\text{RLONG} = \text{COR} * \text{RRLM}$$

and the sol-air temperature is:

$$\text{TSOL} = \text{TDRY} + (\text{ALPHA} * \text{RAD} - \text{EMISS} * \text{RLONG}) / \text{HSO}$$

### 5.A10.9 Solar load imposed by the glazing

Appendix 5.A11 describes the way the admittance method calculates the transmission and absorption of solar radiation within a glazing system. That appendix includes the calculation of the mean and alternating solar gain factors. These factors are only intended to be used in 'hand' calculations; the cooling load calculation makes use of the appropriate value for each hour of the day. That is, the glazing system properties are determined as a function of the solar angle of incidence (section 5.A10.6.2).

Appendix 5.A11, section 5.A11.3, describes how to determine the transmitted and absorbed radiation and calculate the gain to the environmental node and, where internal blinds are used, the air node. The admittance method requires the following to be done at each hour of the day (only between sunrise and sunset, see section 5.A10.5.3).

The following input data are required:

Glass and blind properties including cavity, internal and external thermal resistances (see chapter 3, section 3.6).

Dimensions of the glazed surface (window).

\* <http://www.cibse.org/Guide-A/pdfs>

Dimensions and position of any shading device relative to the window.

ORIEN	Window orientation (radians: North 0 or $2\pi$ )
SLOPE	Angle of window to horizontal (radians: flat roof 0, vertical wall $\pi/2$ )
DIRECT	Direct radiation incident upon an exposed surface ( $\text{W}\cdot\text{m}^{-2}$ )
SKYDIF	Sky diffuse radiation incident on a surface ( $\text{W}\cdot\text{m}^{-2}$ )
GRDREF	Ground reflected radiation incident on a surface ( $\text{W}\cdot\text{m}^{-2}$ )
ANGINC	Solar angle of incidence (radians)

Calculated values (hourly):

QTRANS	Total transmitted solar radiation
QGE	Solar gain to the environmental node
QGA	Solar gain to the air node

For each glazed surface:

— If there are external shading devices other than blinds, determine the amount of shadow created by the device. The calculation of shade is not given here. The calculation of the effect of shade on the performance of a window can be treated at many levels ranging from the simplistic (and probably conservative) approach described here to taking full account of the relationship between the shade, window and sky vault, the reflections of solar radiation within the shading system and the temperature of the shades. The approach here is to assume that the whole window is exposed to diffuse radiation with the direct intensity reduced by the shade fraction.

— Where blinds are fitted determine if they are lowered. This may be a simple schedule or at a particular solar intensity. The cooling load tables assume the blinds are lowered if the intensity of direct radiation on the façade was greater than  $200 \text{ W}\cdot\text{m}^{-2}$ .

— From the solar angle of incidence and the properties of the glazing calculate the transmission coefficient for direct (TAUD) and diffuse radiation (TAUd) and the absorption coefficient for each element in the glazing system.

AGLASS = total window area

AS = area of the window that is in shade

$$\text{QTRANS} = \text{DIRECT} \cdot \text{TAUD} \cdot (\text{AGLASS} - \text{AS}) + \text{AGLASS} \cdot (\text{SKYDIF} + \text{GRDREF}) \cdot \text{TAUD}$$

— If QGED and QGE<sub>d</sub> are the loads at the environmental node for direct and diffuse radiation respectively (determined following section 5.A11.3), and similarly for the air point loads QGAD and QGAd, the load at the environmental node is:

$$\text{QGE} = (\text{AGLASS} - \text{AS}) \cdot \text{QGED} + \text{AGLASS} \cdot \text{QGE}_d$$

$$\text{QGA} = (\text{AGLASS} - \text{AS}) \cdot \text{QGAD} + \text{AGLASS} \cdot \text{QGAd}$$

## 5.A10.10 Calculation of the solar component of the gain

The following data are required.

Surface areas:

SFBAR	Mean surface factor
SFDEL	Mean surface factor delay

The solar loads imposed by the glazing (5.A10.9):

QTRANS	Total transmitted solar radiation for each hour of the day
QGE	Solar gain to the environmental node for each hour of the day.
QGA	Solar gain to the air node for each hour of the day.

Calculated values:

QSESW	The swing in solar cooling load at the environmental node at each hour of the day
QSEBAR	The daily mean solar cooling load at the environmental node
QSASW	The swing in solar cooling load at the air node at each hour of the day
QSABAR	The daily mean solar cooling load at the air node

Carry out the following summations over the day.

$$\text{QTBAR} = \sum \text{QTRANS} / 24$$

$$\text{QSEBAR} = \sum \text{QGE} / 24$$

$$\text{QSABAR} = \sum \text{QSA} / 24$$

The direct solar gain must be absorbed by the room surfaces before it can contribute to the heat load in the room. Due to thermal storage within the surfaces of the room there will be a delay and 'smoothing out' of the direct gain. In the admittance method this is quantified by the 'surface factor' and the associated delay.

The swing in the transmitted load at hour  $H$  is that due to the radiation transmitted at:

$$\text{Hdel} = \text{H} - \text{SFDEL}$$

The swing in the solar gain at the environmental node at hour,  $H$  is:

$$\text{QSESW}(H) = (\text{QGE}(H) - \text{QSEBAR}) + \text{SFBAR} \cdot (\text{QTRANS}(H_{\text{del}}) - \text{QTBAR})$$

The swing in the load at the air node at hour  $H$  is:

$$\text{QSASW}(H) = \text{QGA}(H) - \text{QSABAR}$$

## 5.A10.11 Calculation of the ventilation component of the gain

The following data are required:

TDRY	External dry bulb temperature
------	-------------------------------

TDES	Internal design temperature
CV	Ventilation conductance (5.A10.4)

FRG	Radiant fraction (0 = 100% convective)
-----	--

Calculated values:

QVENTSW	Swing in ventilation gain at the air node.
QVENTB	Mean ventilation load.

Calculated values:

QGASW	Swing in internal gain at the air node
QGAB	Mean internal gain at the air node
QGESW	Swing in internal gain at the environmental node
QGEB	Mean internal gain at the environmental node

The ventilation load at hour  $H$  is:

$$QVENT(H) = CV * (TDRY(H) - TDES)$$

$$QVENTB = \sum QVENT / 24$$

$$QVENTSW(H) = QVENT(H) - QVENTB$$

For each hour ( $H$ ):

$$\text{Convective load: } QC = QGA(H) * (1 - FRG)$$

$$\text{Radiant load: } QR = QGA(H) * FRG$$

### 5.A10.12 Calculation of the conduction component of the gain

The following data are required.

For each surface:

A	Area (opaque and glazed)
U	$U$ -value
D	Decrement factor
DL	Time delay associated with decrement factor
TSOL	Hourly sol-air temperature for each external surface
TSBAR	Daily mean sol-air temperature for each external surface
TDES	Internal design temperature

The load at the air node:

$$QGA(H) = QC - 0.5 * QR$$

The load at the environmental node is:

$$QGE(H) = 1.5 * QR$$

The mean loads are:

$$QGAB = \sum QGA / 24$$

$$QGEB = \sum QGE / 24$$

The swing in load is:

$$QGASW(H) = QGA(H) - QGABAR$$

$$QGESW(H) = QGE(H) - QGEBAR$$

Calculated values:

QCSW	The swing in conduction gain at the environmental node
QCSB	The mean conduction gain at the environmental node

#### Mean conduction gain

For each hour ( $H$ ) of the day, calculate for each external surface ( $N$ ) and sum:

$$QC(H) = U(N) * A(N) * (TSOL(H, N) - TDES)$$

$$QCSB = \sum QC / 24$$

The swing in gain is due to the gain that occurred  $DL$  hours before the current hour, that is:

$$HDEL = H - DL$$

$$QCSW(H) = U(N) * A(N) * D(N) * (TSOL(HDEL, N) - TSBAR(N))$$

### 5.A10.13 Calculation of the internal gain

The following data are required.

QGI	Hourly value of the internal gain ( $W$ )
-----	---

### 5.A10.14 Calculation of the total gain and the solar cooling load for 24-hour plant operation

The calculation here is for control by air temperature or operative temperature. In the case of the cooling load tables only the solar load is considered. The air change rate is used in the calculation of the non-dimensional parameters only.

The following data are required.

Daily mean values of loads at the environmental node:

QSEBAR	Daily mean solar cooling load (5.A10.10)
QCSB	Mean conduction gain (5.A10.12)
QGEB	Mean internal gain (5.A10.13)

Daily mean values at the air node:

QSABAR	Daily mean solar cooling load (5.A10.10)
QVENTB	Mean ventilation load (5.A10.11)
QGAB	Mean internal gain (5.A10.13)

Hourly swing in load at the environmental node:



QSESW	Swing in solar cooling load (5.A10.10)
QCSW	Swing in conduction gain (5.A10.12).
QGESW	Swing in internal gain (5.A10.13)

Hourly swing in load at the air node:

QSASW	Swing in solar cooling load (5.A10.10).
QVENTSW	Swing in ventilation gain (5.A10.11).
QGASW	The swing in internal gain (5.A10.13).

Non-dimensional parameters (5.A10.4):

FU	FY
F1C	F1AY
F2C	F2AY
F1A	F1CY
F2A	F2CY

Calculated values:

QPBARDaily mean plant load	
QPSWG	Hourly swing in the plant load
QPLANT	Hourly cooling load

The following sums are required:

QGENVB	Sum of all mean gains to the environmental node
QGAIRB	Sum of all mean gains to the air node
QGENVS	Sum of all swings in gain at the environmental node for each hour ( $H$ ) of the day
QGAIRS	Sum of all swings in gain at air node for each hour ( $H$ ) of the day.

Control by operative temperature

$$QPBAR = F1C * QGENVB + F2C * QGAIRB$$

$$QPSWG(H) = F1CY * QGENVS(H) + F2CY * QGAIRS(H)$$

Control by air temperature

$$QPBAR = F1A * QGENVB + F2A * QGAIRB$$

$$QPSWG(H) = F1AY * QGENVS(H) + F2AY * QGAIRS(H)$$

Hourly cooling load

$$QPLANT(H) = QPBAR + QPSWG(H)$$

### 5.A10.15 Calculation of total gain for intermittent plant operation

The following data are required:

QPLANT	Hourly cooling load for 24-hour plant operation (5.A10.14)
--------	--

PLNTON	Time plant switched on
PLNTOFF	Time plant switched off
DOUTPT	Intermittency correction factor (5.A10.4)

Sum the cooling load for all hours ( $H$ ) for which the plant is off ( $QB$ ). This is the sum of  $QPLANT$  when  $H$  is less than  $PLNTON$  or  $H$  is greater than  $PLNTOFF$ .

The cooling load for each hour for which the plant is switched on is:

$$QPLANTI(H) = QPLANT(H) + QB * DOUTPT$$

Otherwise:

$$QPLANTI(H) = 0.0$$

### 5.A10.16 Example calculation

This example uses an implementation of the algorithm described in this appendix to perform the manual cooling load calculation described in Appendix 5.A6.3. In both cases the space geometry, construction, internal gains and climatic data are the same (see Figure 5.A10.1 and Table 5.A10.1).

Because the algorithm determines the solar gain by the methodology described in Appendix 5.A11, it was necessary to devise a glazing system that would have a  $U$ -value of  $2.2 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$  and a  $G$ -value of 0.6. To do this the inner glazing element was assigned the optical properties of a typical low emissivity glass (transmission coefficient 0.6, reflectivity 0.27 and absorptivity 0.18).

The thermal resistance of the cavity was set to the value necessary to achieve a  $U$ -value of  $2.2 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$  and the outer pane was taken as clear glass, the transmission being adjusted to achieve a  $G$ -value close to 0.6. The calculation of cooling load for control to both operative and air temperature with 24-hour plant operation and 8-hour plant is described in Tables A5.10.2 to A5.10.4. The resulting design values, which all occur between hours of 15:00 and 16:00, are:

- *Control to operative temperature:* 2391 W for continuous operation and 3060 W for 8-hour plant operation.
- *Control to air temperature:* 1890 W for continuous operation and 2633 W for 8-hour plant operation.

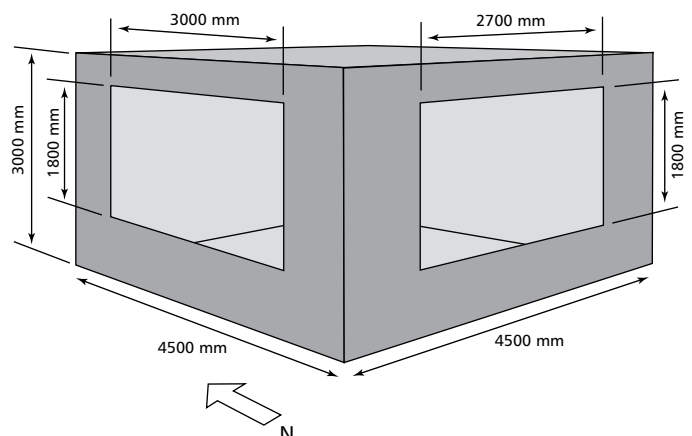


Figure 5.A10.1 Example calculation: south facing office module

**Table 5.A10.1** Example calculation: dimensions and properties

Surface no.	Area / m <sup>2</sup>	Orientation / degree	U-value / W·m <sup>-2</sup> ·K <sup>-1</sup>	Admittance		Decrement factor		Surface factor		Solar absorptivity	G-value
				Y-value / W·m <sup>-2</sup> ·K <sup>-1</sup>	Time lead / h	Value	Time lag / h	Value	Time lag / h		
<b>Opaque surfaces</b>											
1 (ext. wall)	8.6	180 (S)	0.35	0.94	4.02	0.99	1	0.95	1	0.6	—
2 (ext. wall)	8.6	270 (W)	0.35	4.4	1.79	0.3	9	0.588	2	0.6	—
3 (int. wall)	13.5	—	1	4.13	3	0.1	3	0.5	2	—	—
4 (int wall)	13.5	—	1	4.13	3	0.1	3	0.5	2	—	—
5 (ceiling)	20.3	—	1	2	3	0.1	3	0.5	2	—	—
6 (floor)	20.3	—	1	5.3	6	0.1	3	0.5	2	—	—
<b>Windows</b>											
1	4.37	180 (S)	2.2	2.2	0	1	0	0.9	0	—	0.6
2	4.86	270 (W)	2.2	2.2	0	1	0	0.9	0	—	0.6

Notes: window 1 is in surface 1 and window 2 in surface 2; there are no other external surfaces. The space volume is 60.75 m<sup>3</sup>.

**Table 5.A10.2** Example calculation: dimensionless parameters

Name	Value	Name	Value	Name	Value
FU	0.985	F1AU	0.937	F1C	0.988
FY	0.839	F2AU	1	F2C	1
FV	0.996	F1AY	0.566	F1CY	0.842
FAU	0.941	F2AY	1	F2CY	1
FAY	0.566	F1A	0.941		
FCU	0.988	F2A	1		

**Table 5.A10.3** Example calculation: calculated hourly values

Hour	TDRY	QHdir	QHdif	Ccover	Surface no. 1			Surface no.2		
					Qsnorm	Tsol	Qtrans	Qsnorm	Tsol	Qtrans
0 to 1	12.1	0	0	10	0	11.96	0	0	11.96	0
1 to 2	13.1	0	0	10	0	12.06	0	0	12.06	0
2 to 3	12.4	0	0	10	0	12.21	0	0	12.21	0
3 to 4	12.6	0	0	10	0	11.96	0	0	11.96	0
4 to 5	12.7	0	0	10	0	12.11	0	0	12.11	0
5 to 6	13.4	0	2	10	0	12.51	0	0	12.51	0
6 to 7	13.9	9	36	10	32.4	13.89	12.5	17.3	13.53	7.5
7 to 8	15.0	71	99	10	160.1	17.75	64.6	52.2	15.16	22.7
8 to 9	16.9	127	152	10	288	22.32	126.4	82	17.38	35.7
9 to 10	18.0	300	178	10	564.4	30.46	256	111.2	19.58	48.4
10 to 11	19.4	347	207	10	665.2	34.13	301.8	129.1	21.26	56.2
11 to 12	20.6	387	231	10	750	37.46	340.2	144	22.92	62.7
12 to 13	21.4	386	230	10	747.4	38.4	339.1	242.2	26.27	88.2
13 to 14	21.9	358	212	10	686.6	37.59	311.4	426.7	31.35	178.6
14 to 15	22.7	311	184	10	586.3	35.83	265.9	584.5	35.79	265.1
15 to 16	22.7	229	138	10	417.1	32.17	183.5	655.2	37.88	298.2
16 to 17	22.3	145	89	10	243	27.79	95.3	676.4	38.19	308.7
17 to 18	20.5	53	34	10	61.2	22.33	17.4	541.6	33.86	248.1
18 to 19	18.4	0	5	10	0	18.91	0	0	18.91	0
19 to 20	17.2	0	0	10	0	17.26	0	0	17.26	0
20 to 21	17.4	0	0	10	0	16.76	0	0	16.76	0
21 to 22	16.8	0	0	10	0	16.56	0	0	16.56	0
22 to 23	16.0	0	0	10	0	15.86	0	0	15.86	0
23 to 0	12.9	0	0	10	0	13.91	0	0	13.91	0
Mean	17.1	113	75	10	216.8	21.76	96.4	87.72	66.7	67.50

**Table 5.A10.4** Example calculation: calculated hourly values

Hour	Swings								Hourly values for control on stated temperature				
	QSESW	QCSW	QGESW	QGENVS	QSASW	QVENT	QSASW	QGAIRS	Operative temp.		Air temp.		
									QPLANT	QPLANTI	QPLANT	QPLANTI	
0 to 1	-620.2	-102.1	-90.6	-813	0	-9.3	-30.2	-40	-150.5	0	-333.8	0	
1 to 2	-620.2	-106	-90.6	-817	0	-9.1	-30.2	-39	-147.5	0	-331.8	0	
2 to 3	-620.2	-106.6	-90.6	-817	0	-8.8	-30.2	-39	-147.5	0	-331.8	0	
3 to 4	-620.2	-124.7	-90.6	-836	0	-9.3	-30.2	-40	-131.5	0	-321.8	0	
4 to 5	-620.2	-124	-90.6	-835	0	-9	-30.2	-39	-132.5	0	-321.8	0	
5 to 6	-620.2	-115.8	-90.6	-827	0	-8.2	-30.2	-38	-140.5	0	-327.8	0	
6 to 7	-600.6	-102.5	-90.6	-794	0	-7	-30.2	-37	-168.5	0	-347.8	0	
7 to 8	-533.6	-82.6	-90.6	-707	0	-5.4	-30.2	-36	-243.5	-882	-397.8	-1107	
8 to 9	-399.1	-41.7	181.3	-260	0	-2.3	60.4	58	-714.5	-1033	-744.8	-1209	
9 to 10	-85.9	1.5	181.3	97	0	0.7	60.4	61	-1017.5	-1655	-949.8	-1658	
10 to 11	167	52.7	181.3	401	0	3.2	60.4	64	-1275.5	-1914	-1124.8	-1833	
11 to 12	592.1	90.8	181.3	864	0	5.9	60.4	66	-1668.5	-2307	-1388.8	-2098	
12 to 13	757.9	121.5	181.3	1061	0	7.9	60.4	68	-1835.5	-2474	-1501.8	-2211	
13 to 14	976.8	137.8	181.3	1296	0	9.2	60.4	70	-2035.5	-2673	-1636.8	-2345	
14 to 15	1074.7	148.8	181.3	1405	0	10.5	60.4	71	-2128.5	-2766	-1699.8	-2408	
15 to 16	1236.2	152.3	181.3	1570	0	11.3	60.4	72	-2268.5	-2906	-1793.8	-2502	
16 to 17	1285.6	138.1	-90.6	1333	0	10.9	-30.2	-19	-1977.5	-2935	-1568.8	-2522	
17 to 18	1022	103.9	-90.6	1035	0	8.7	-30.2	-21	-1724.5	0	-1397.8	0	
18 to 19	549	49	-90.6	507	0	4.8	-30.2	-25	-1276.5	0	-1095.8	0	
19 to 20	160.1	6.2	-90.6	76	0	1.4	-30.2	-29	-909.5	0	-847.8	0	
20 to 21	-620.2	-7.6	-90.6	-718	0	0.4	-30.2	-30	-239.5	0	-397.8	0	
21 to 22	-620.2	-10.2	-90.6	-721	0	0	-30.2	-30	-237.5	0	-395.8	0	
22 to 23	-620.2	-20.5	-90.6	-731	0	-1.4	-30.2	-32	-227.5	0	-388.8	0	
23 to 0	-620.2	-58.3	-90.6	-769	0	-5.4	-30.2	-36	-191.5	0	-362.8	0	
Mean	937	-159	91	869	0	-14	30	16	-874.5	-897.708	-833.8	-828.875	
									Total:	-20990	-21545	-20010.2	-19893

### 5.A10.17 Solar cooling load tables

The UK cooling load tables (Tables 5.16(a) to (n) and 5.17(a) to (n)) and the similar tables for latitudes from 0–60° N/S available from the CIBSE website\* were calculated using the algorithm described in this appendix with no internal, conduction or ventilation gains.

(a) For the tables relating to unshaded situations, values of parameters used for calculating the non-dimensional factors were as follows:

- *module location*: intermediate floor with one exposed surface
- *module dimensions*: (4.8 × 4.8 × 2.7) m
- *glazed percentage*: 40% of external wall
- *properties of surfaces of module*: see Table 5.A10.5.

For both cases (i.e. fast and slow thermal response), a relatively well-sealed facade was assumed, with an infiltration rate of 0.25 h<sup>-1</sup>.

- (b) Glazing properties were as given in Table 5.A10.6.
- (c) Shading: the tables relating to shaded situations, a generic shading device having 20% transmission and 40% reflection was assumed, see Table 5.A10.6. The shading device was assumed to operate when direct radiation on the façade was greater than 200 W·m<sup>-2</sup>.

## References for Appendix 5.A10

CIBSE (2002) *Weather, solar and illuminance data* CIBSE Guide J (London: Chartered Institution of Building Services Engineers)

Harrington-Lynn J (1974) *The admittance procedure: variable ventilation Building Serv. Eng.* 42 199–200 (November 1974)

White AJ, Holmes MJ, Hacker JN, De Saulles T and Crawley N (2012) 'Passive Design Assistant — a tool to elucidate the principles of passive design' *Proc. CIBSE/ASHRAE Tech. Symp.*, 2012

\* <http://www.cibse.org/Guide-A/pdfs>

**Table 5.A10.5** Properties of surfaces for module used for determination of cooling load tables

Surface	Slow thermal response				Fast thermal response			
	U-value / W·m <sup>-2</sup> ·K <sup>-1</sup>	Y-value / W·m <sup>-2</sup> ·K <sup>-1</sup>	Surface factor, F	Time lag ψ / h	U-value / W·m <sup>-2</sup> ·K <sup>-1</sup>	Y-value / W·m <sup>-2</sup> ·K <sup>-1</sup>	Surface factor, F	Time lag ψ / h
Glass	3.0	3.0	—	—	3.0	3.0	—	—
Wall	0.45	5.5	0.5	2	0.45	2.0	0.8	1

**Table 5.A10.6** Transmission, absorption and reflection components and emissivities for generic glass and blind combinations

Description	Shortwave radiation (proportions of total)			Longwave emissivity	
	Transmitted	Reflected 1	Reflected 2	Surface 1	Surface 2
<b>Glass:</b>					
— clear	0.789	0.072	0.072	0.837	0.837
— low emissivity*	0.678	0.091	0.108	0.837	0.17
— absorbing	0.46	0.053	0.053	0.837	0.837
— reflecting (high performance)*	0.39	0.31	0.45	0.837	0.025
<b>Slatted blind†:</b>					
— reflecting	0.0	0.60	0.40	0.80	0.80
— absorbing	0.0	0.80	0.20	0.80	0.80
‘Generic’ blind	0.20	0.40	0.40	0.80	0.80

\* Asymmetric glass properties

## Appendix 5.A11: Derivation of solar gain factors

### 5.A11.1 Notation

Symbols used in this appendix are as follows.

$a$	Fraction of incident energy absorbed by thickness $L$ (mm) of glass
$A$	Absorption coefficient
$A'$	Absorption coefficient for double glazing
$A''$	Absorption coefficient for triple glazing
$A_D$	Absorption coefficient for direct radiation
$A_d$	Absorption coefficient for diffuse radiation
$A_{dg}$	Absorption coefficient for ground reflected radiation
$A_{ds}$	Absorption coefficient for sky diffuse radiation
$C_1, C_2$	Configuration factors for slatted blinds
$D$	Slat thickness (mm)
$F$	Surface factor
$h$	Solar altitude (degree)
$H$	Transmittance factor
$I$	Incident solar irradiance ( $\text{W}\cdot\text{m}^{-2}$ )
$j$	Number of surface
$k$	Glass extinction coefficient
$L$	Glass thickness (mm)
$M$	Width of slat illuminated (mm)
$n$	Total number of surfaces
$R$	Reflection coefficient
$R'$	Reflection coefficient for double glazing
$R''$	Reflection coefficient for triple glazing
$r$	Ratio of incident beam to reflected beam at air/glass interface
$r_{//}$	Ratio of incident beam to reflected beam at air/glass interface for radiation polarised parallel to the plane of incidence
$r_{\perp}$	Ratio of incident beam to reflected beam at air/glass interface for radiation polarised perpendicular to the plane of incidence
$R_D$	Reflection coefficient for direct radiation
$R_d$	Reflection coefficient for diffuse radiation
$R_{dg}$	Reflection coefficient for ground reflected radiation
$R_{ds}$	Reflection coefficient for sky diffuse radiation
$R_{se}$	External surface resistance ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$R_{si}$	Internal surface resistance ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )
$T$	Transmission coefficient
$T'$	Transmission coefficient for double glazing
$T''$	Transmission coefficient for triple glazing
$T_D$	Transmission coefficient for direct radiation
$T_d$	Transmission coefficient for diffuse radiation
$T_{dg}$	Transmission coefficient for ground reflected radiation
$T_{ds}$	Transmission coefficient for sky diffuse radiation
$T_n$	Transmission coefficient at normal incidence
$t$	Time (h)
$W$	Slat width (m)
$\alpha$	Absorptivity (thermal shortwave radiation)
$\beta$	Profile angle (degree)
$\gamma$	Wall azimuth (degree)
$\gamma_s$	Wall-solar azimuth (degree)
$\zeta_i$	Angle of incidence (degree)
$\zeta_r$	Angle of refraction (degree)
$\theta$	Temperature ( $^{\circ}\text{C}$ )
$\mu$	Refractive index of glass (= 1.52)
$\sigma_v$	Vertical shadow angle (degree)
$\Phi$	Room gain ( $\text{W}\cdot\text{m}^{-2}$ )

$\Phi_a$	Room gain to air node ( $\text{W}\cdot\text{m}^{-2}$ )
$\Phi_{at}$	Room gain to air node at time $t$ ( $\text{W}\cdot\text{m}^{-2}$ )
$\Phi_e$	Room gain to environmental node ( $\text{W}\cdot\text{m}^{-2}$ )
$\Phi_{et}$	Room gain to environmental node at time $t$ ( $\text{W}\cdot\text{m}^{-2}$ )
$\Phi_t$	Room gain at time $t$ ( $\text{W}\cdot\text{m}^{-2}$ )
$\phi$	Solar azimuth (degree)
$\psi$	Slat angle (degree)
$\omega$	Time lag associated with surface factor (h)

Where required additional subscripts 'A', 'R' and 'T' indicate gains due to absorbed, reflected and transmitted components of radiation, respectively.

### 5.A11.2 Response of room to solar radiation

Shortwave solar radiation incident upon a window will be reflected, absorbed in the glazing elements or directly transmitted into the space beyond the window. The absorbed radiation will increase the temperature of the glazing and is therefore both a longwave radiant heat gain and a convective gain to the space. In terms of the simple (dynamic) model these gains are considered to enter the model at the environmental node. If internal blinds are present, there will be an increase in the convective portion of the gain which enters the model at the air node. Transmitted radiation must be absorbed at the room surfaces before it can become a heat gain to the space. With the exception of any shortwave radiation that passes directly out of the space by transmission through glazed surfaces, all the radiation entering the space is absorbed at the room surfaces or within the furnishings.

Once absorbed, the radiation warms the surfaces and, after a time delay, enters the space at the environmental node by means of convection and radiation.

For the purposes of the simple (dynamic) model, the room gain is divided into a 24-hour mean component and an hourly cyclic component.

For any given source:

$$\bar{\Phi} = (1/24) \sum_{t=1}^{t=24} \Phi_t \quad (\text{A11.1})$$

and:

$$\tilde{\Phi}_t = \Phi_t - \bar{\Phi} \quad (\text{A11.2})$$

In the case of the simple (dynamic) model the gain will be either to the environmental node only or to both the environmental and air nodes. The gain to the environmental node from transmitted radiation is:

$$\Phi_{eTt} = \bar{\Phi}_{eT} + F \tilde{\Phi}_{t eT(t-\omega)} \quad (\text{A11.3})$$

$$\tilde{\Phi}_{eTt} = \Phi_{eTt} - \bar{\Phi}_{eT} \quad (\text{A11.4})$$

$$\bar{\Phi}_{eT} = (1/24) \sum_{t=1}^{t=24} \Phi_{eTt} \tag{A11.5}$$

$$\Phi_{eTt} = T I_t \tag{A11.4}$$

where  $\Phi_{eT}$  is the overall gain to the environmental node from transmitted radiation at time  $t$ .

However, in practice, direct and diffuse transmitted radiation must be treated separately.

The gain to the environmental node due to conduction and radiation from the inner surface of the glazing is:

$$\Phi_{eAt} = \sum_{j=1}^{j=n} (H_{ej} A_j I_t) \tag{A11.7}$$

where  $A$  is the component of the radiation absorbed by the glass, subscript  $j$  denotes the number of the glazing element and  $n$  is the total number of glazing elements within the window system.

Thus the total gain to the environmental node is:

$$\Phi_{et} = \Phi_{eTt} + \Phi_{eAt} \tag{A11.8}$$

Additionally, if there is an internal blind, the gain to the air node is:

$$\Phi_{aAt} = \sum_{j=1}^{j=n} (H_{aj} A_j I_t) \tag{A11.9}$$

Thus the total gain to the air node is:

$$\Phi_{at} = \Phi_{aAt} \tag{A11.10}$$

To simplify the calculation of these gains, solar gain factors are used. These are the ratios of the components of the gain to the incident solar radiation. The room load has both steady state and cyclic components and the space gains are to the environmental and, possibly, the air nodes. Additionally, the surface factor depends on the response time of the space. To calculate the solar gain factors, typical values are taken, as follows:

- for slow response space:  $F = 0.5$ ; time delay = 2 h
- for a fast response space:  $F = 0.8$ ; time delay = 1 h

The solar gain factors are as follows:

$$\bar{S}_e = \bar{\Phi}_e / \bar{I} \tag{A11.11}$$

$$\tilde{S}_{et} = \tilde{\Phi}_{et} / \tilde{I}_t \tag{A11.12}$$

$$\bar{S}_a = \bar{\Phi}_a / \bar{I} \tag{A11.13}$$

$$\tilde{S}_{at} = \tilde{\Phi}_{at} / \tilde{I}_t \tag{A11.14}$$

Solar gain factors for generic glass and blind combinations are given in Table 5.20 (repeated here as Table 5.A11.1). These have been calculated using banded solar radiation data for Kew (1959–1968) incident on a south-west facing vertical window (see chapter 2). The transmission (T), absorption (A) and reflection (R) components (for thermal

**Table 5.A11.1** Solar gain factors and shading coefficients for generic glazing/blind combinations

Description (inside to outside)	Solar gain factor at environmental node†			Solar gain factor at air node		Shading coefficient, $S_c$	
	$\bar{S}_e$	$\tilde{S}_{el}$	$\tilde{S}_{eh}$	$\bar{S}_a$	$\tilde{S}_a$	Shortwave	Longwave
<b>Single glazing/blind combinations:</b>							
— clear glass	0.76	0.66	0.50	—	—	0.91	0.05
— absorbing glass	0.61	0.54	0.44	—	—	0.53	0.19
— absorbing slats/clear	0.43	0.44	0.44	0.17	0.18	—	—
— reflecting slats/clear	0.35	0.32	0.31	0.12	0.12	—	—
— ‘generic’ blind/clear	0.34	0.33	0.29	0.11	0.11	—	—
<b>Double glazing/blind combinations:</b>							
— clear/clear	0.62	0.56	0.44	—	—	0.70	0.12
— clear/reflecting	0.36	0.32	0.26	—	—	0.37	0.08
— low emissivity/clear	0.62	0.57	0.46	—	—	0.62	0.18
— low emissivity/absorbing	0.43	0.38	0.32	—	—	0.36	0.15
— low emissivity/clear/‘generic’ blind	0.15	0.14	0.11	—	—	—	—
— absorbing slats/clear/clear	0.34	0.36	0.37	0.18	0.21	—	—
— absorbing slats/clear/reflecting	0.19	0.19	0.19	0.12	0.13	—	—
— absorbing slats/low emissivity/clear	0.33	0.35	0.35	0.21	0.23	—	—
— absorbing slats/low emissivity/absorbing	0.22	0.22	0.22	0.16	0.17	—	—
— reflecting slats/clear/clear	0.28	0.29	0.26	0.15	0.16	—	—
— reflecting slats/clear/reflecting	0.17	0.16	0.16	0.10	0.10	—	—
— reflecting slats/low emissivity/clear	0.28	0.27	0.26	0.18	0.20	—	—
— reflecting slats/low emissivity/absorbing	0.18	0.17	0.17	0.14	0.15	—	—
— ‘generic’ blind/low emissivity/clear	0.29	0.29	0.27	0.17	0.18	—	—
<b>Triple glazing:</b>							
— clear/clear/clear	0.52	0.49	0.40	—	—	0.55	0.17
— clear/clear/absorbing	0.37	0.35	0.29	—	—	0.33	0.15
— clear/clear/reflecting	0.30	0.28	0.23	—	—	0.30	0.09
— clear/low emissivity/clear	0.53	0.50	0.42	—	—	0.50	0.21

† For  $\tilde{S}_e$ , subscripts ‘l’ and ‘h’ denote thermally ‘lightweight’ and ‘heavyweight’ buildings, respectively

Note: shading coefficients for windows with slatted blind or windows with inner blind are not given since these not compatible with the properties of plain glass

**Table 5.A11.2** Transmission, absorption and reflection components and emissivities for generic glass and blind combinations

Description	Shortwave radiation (proportions of total)			Longwave emissivity	
	Transmitted	Reflected 1	Reflected 2	Surface 1	Surface 2
Glass:					
— clear	0.789	0.072	0.072	0.837	0.837
— low emissivity*	0.678	0.091	0.108	0.837	0.17
— absorbing	0.46	0.053	0.053	0.837	0.837
— reflecting (high performance)*	0.39	0.31	0.45	0.837	0.025
Slatted blind†:					
— reflecting	0.0	0.60	0.40	0.80	0.80
— absorbing	0.0	0.80	0.20	0.80	0.80
‘Generic’ blind	0.20	0.40	0.40	0.80	0.80

\* Asymmetric glass properties

shortwave radiation) and emissivities (for thermal longwave radiation) for the generic glass and blind types used in calculating the solar gain factors are given in Table 5.A11.2.

Effectively, solar gain factors can only be calculated by means of a computer program. The following sections describe the basis of the calculation procedure.

### 5.A11.3 Transmission, absorption and reflection for direct solar radiation

#### Clear glass

For clear glass the transmission, absorption and reflection (TAR) coefficients can be derived theoretically (Jones, 1980).

The angle of refraction is obtained from the angle of incidence using Snell’s Law:

$$\zeta_r = \arcsin(\sin \zeta_i / \mu) \quad (\text{A11.15})$$

The reflected beams for radiation polarised parallel to and perpendicular to the plane of incidence are determined using Fresnel’s formula:

$$r_{\parallel} = \frac{\tan^2(\zeta_i - \zeta_r)}{\tan^2(\zeta_i + \zeta_r)} \quad (\text{A11.16})$$

$$r_{\perp} = \frac{\sin^2(\zeta_i - \zeta_r)}{\sin^2(\zeta_i + \zeta_r)} \quad (\text{A11.17})$$

As the angle of incidence approaches 0 (i.e. normal incidence):

$$\tan \zeta_i > \sin \zeta_i > \zeta_i \quad (\text{A11.18})$$

hence:

$$r_{\parallel} > r_{\perp} > \frac{(\mu - 1)^2}{(\mu + 1)^2} \quad (\text{A11.19})$$

This is a useful result as it enables the calculation of the extinction coefficient ( $k$ ) if the transmission at normal incidence ( $T_n$ ) is known. The extinction coefficient is a non-linear function of the glass thickness ( $L$ ) and is related to the transmission coefficient by:

$$T_n = \frac{(1 - r)^2 \exp(-kL)}{1 - r^2 \exp(-2kL)} \quad (\text{A11.20})$$

For the beam polarised parallel to the plane of incidence, the fraction of incident energy absorbed for each beam is calculated as follows:

$$a_{\parallel} = 1 - \exp(-kL / \cos \zeta_r) \quad (\text{A11.21})$$

and similarly for the perpendicularly polarised beam.

The transmitted, absorbed and reflected coefficients are calculated separately for each beam (i.e. parallel and perpendicularly polarised) and the average taken to give the overall coefficients. For the beam polarised parallel to the plane of incidence:

$$T_{D\parallel} = \frac{(1 - r)^2 (1 - a_{\parallel})}{1 - r^2 (1 - a_{\parallel})^2} \quad (\text{A11.22})$$

$$A_{D\parallel} = \frac{a_{\parallel} (1 - r) [1 + r (1 - a_{\parallel})]}{1 - r^2 (1 - a_{\parallel})^2} \quad (\text{A11.23})$$

$$R_{D\parallel} = \frac{r (1 - r)^2 (1 - a_{\parallel})}{1 - r^2 (1 - a_{\parallel})} + r \quad (\text{A11.24})$$

and similarly for the perpendicularly polarised beam.

Therefore:

$$T_D = 1/2 (T_{D\parallel} + T_{D\perp}) \quad (\text{A11.25})$$

and similarly for the absorption and reflection coefficients.

Note that since the transmitted, absorbed and reflected components add up to unity, only two need be calculated, the third being obtained by subtraction.

### Reflecting and other glasses

The characteristics of such glasses differ from those for plain glass and therefore must be obtained from the manufacturers. If the characteristics are supplied as a graph of  $\tau$  coefficients against angle of incidence, the appropriate values can be read-off directly or by curve-fitting techniques.

### Slatted blinds

The analysis is the same for both horizontal and vertical slatted blinds. Radiation may be transmitted into a room by the following paths (Parmelee and Vild, 1953).

- *direct*: i.e. passes through the blind without touching any surface; may be zero
- *reflected (1)*: i.e. passes through the blind after one reflection from the slat surface which is directly irradiated by the sun
- *reflected (2)*: i.e. passes through the blind after undergoing any number of reflections, the final reflection being from the slat surface opposite the one directly illuminated by the sun
- *reflected (3)*: i.e. passes through the blind after undergoing any number of reflections, the final reflection being from the one directly illuminated by the sun.

In order to calculate these components, up to five configuration factors are required, each of which depends on the blind geometry. The number of factors needed depends on whether all or only part of the slat is illuminated.

The amount of a slat that is illuminated (i.e. not shaded by the slat above it) depends on the geometry of the blind and the 'profile angle'.

The profile angle ( $\beta$ ) is the angle that the direct radiation beam makes with the blind in a vertical plane perpendicular to the plane of the window. For horizontal slatted blinds on a vertical window, the profile angle is the vertical shadow angle:

$$\beta = \sigma_v = \arctan(\tan h \sec \gamma_s) \quad (\text{A11.26})$$

For vertical slatted blinds on a vertical window, the profile angle is the wall-solar azimuth:

$$\beta = \gamma_s = \phi - \gamma \quad (\text{A11.27})$$

In the following analysis, it is assumed that the radiation is incident on the upper surface of the slat. The width of slat that is illuminated is calculated from:

$$M = \min \left( W, \frac{D \cos \beta}{\sin(\beta + \psi)} \right) \quad (\text{A11.28})$$

The configuration factors are calculated as follows.

Radiation that is reflected by the lower slat and passes into the room when the whole width is illuminated ( $C_1$ ):

$$C_1 = \frac{1}{2} \{1 + (D/W) - [1 + (D^2/W^2) + (2D \sin \psi / W)]^{1/2}\} \quad (\text{A11.29})$$

Radiation that is reflected by the lower slat and intercepted by the upper slat when the whole width is illuminated ( $C_2$ ):

$$C_2 = \frac{1}{2} \{[1 + (D^2/W^2) + (2D \sin \psi / W)]^{1/2} + [1 + (D^2/W^2) - (2D \sin \psi / W)]^{1/2} - (2D/W)\} \quad (\text{A11.30})$$

Radiation reflected by the upper slat which passes into the room ( $C_3$ ):

$$C_3 = \frac{1}{2} \{[1 + (D/W) - [1 + (D^2/W^2) - (2D \sin \psi / W)]^{1/2}\} \quad (\text{A11.31})$$

Radiation reflected by the lower slat, which passes into the room when the lower slat is partially shaded ( $C_4$ ):

$$C_4 = \frac{1}{2} (1 + \{[(W-M)^2/M^2] + (D^2/M^2) + [2(W-M)D \sin \psi / M^2]\}^{1/2} - [(W^2/M^2) + (D^2/M^2) + (2WD \sin \psi / M^2)]^{1/2}) \quad (\text{A11.32})$$

Radiation reflected by the lower slat, which is intercepted by the upper slat when the lower slat is partially shaded ( $C_5$ ):

$$C_5 = \frac{1}{2} \{[(W^2/M^2) + (D^2/M^2) + (2DW \sin \psi / M^2)]^{1/2} - (D/M) + [1 + (D^2/M^2) - (2D \sin \psi / M)]^{1/2} - \{[(W-M)^2/M^2] + (D^2/M^2) + [2(W-M)D \sin \psi / M^2]\}^{1/2}\} \quad (\text{A11.33})$$

If the whole of the lower slat is illuminated and some radiation may pass directly into the room, the TAR coefficients for the blind are calculated from:

$$T_D = 1 - \left( \frac{W \sin(\phi + \psi)}{D \cos \phi} \right) \times \left( 1 - C_1(1-a) - \frac{C_2(1-a)^2 - [C_3 + C_1 C_2(1-a)]}{1 - C_2^2(1-a)^2} \right) \quad (\text{A11.34})$$

$$A_D = \frac{a W \sin(\phi + \psi)}{D \cos \phi [1 - C_2(1-a)]} \quad (\text{A11.35})$$

$$R_D = 1 - A_D - T_D \quad (\text{A11.36})$$



Where part of the lower slat is shaded by the slat above:

$$T_D = C_4(1-a) + \{C_5(1-a)^2 \times \left( \frac{C_3 + C_1 C_2(1-a)}{1 - C_2^2(1-a)^2} \right) \quad (\text{A11.37})$$

$$A_D = a \left( 1 + \frac{C_5(1-a)}{1 - C_2(1-a)} \right) \quad (\text{A11.38})$$

$$R_D = 1 - A_D - T_D \quad (\text{A11.39})$$

### Roller blinds

The properties for roller blinds are not well defined. It is generally sufficient to assume that the TAR coefficients are independent of the angle of incidence and take the values at normal incidence supplied by the manufacturers.

#### 5.A11.4 Transmission, absorption and reflection for sky diffuse and ground reflected radiation

Transmission, absorption and reflection coefficients for glasses and blinds are calculated by considering the direct properties over a range of angles appropriate to the radiation. For glass, the TAR values for sky diffuse and ground reflected radiation are the same since glass has symmetrical properties. The characteristics for roller blinds can be assumed to be the same for direct and diffuse radiation. However, slatted blinds are highly asymmetrical so the two sources of diffuse radiation must be calculated separately.

### Glasses

The standard properties are calculated on the assumption that the glass is exposed to a hemispherical source of uniform radiance therefore the transmission and absorption angles are from 0° to 90°. Mathematically, the expressions for tar could be integrated over this range, i.e.:

$$T_d = \int_0^{90} T_d \xi_i \sin(2 \xi_i) d\xi_i \quad (\text{A11.40})$$

In practice the direct properties are summed for angles of incidence from 2.5° to 87.5° at intervals of 5°, i.e.:

$$T_d = \sum_{\xi=2.5}^{\xi=87.5} \{T_D \xi [\sin^2(\xi_i + 2.5) - \sin^2(\xi_i - 2.5)]\} \quad (\text{A11.41})$$

$A_d$  is calculated similarly and  $R_d$  is obtained by subtraction from unity, see equation A11.36.

### Slatted blinds

The direct properties are summed for profile angles from 5° to 85° at intervals of 10° for sky diffuse radiation. For ground reflected radiation, they are summed from -85° to -5° at intervals of 10° taking into account the configuration factor of the hemispherical radiating source bounded by

profile angles of  $(\beta + 5)^\circ$  and  $(\beta - 5)^\circ$  (Nicol, 1966). Thus, for sky diffuse radiation:

$$T_{ds} = \sum_{\beta=5}^{\beta=85} \{T_D \beta [\sin(\beta + 2.5) - \sin(\beta - 2.5)]\} \quad (\text{A11.42})$$

For ground reflected radiation:

$$T_{dg} = \sum_{\beta=-5}^{\beta=-85} \{T_D \beta [\sin(\beta + 2.5) - \sin(\beta - 2.5)]\} \quad (\text{A11.43})$$

$A_{ds}$  and  $A_{dg}$  are calculated similarly and  $R_{ds}$  and  $R_{dg}$  are obtained by subtraction from unity, see equation A11.36.

#### 5.A11.5 Properties of glass and blind combinations

The properties of multiple layer windows can be calculated from the properties of the individual components. There are many glass types and many permutations; the method of calculation is demonstrated in the following for double and triple glazing using generic glass and blind types.

In the same way that the properties of a single sheet of glass are calculated from the fundamental properties of the glass and an infinite number of inter-reflections at both glass/air interfaces, the properties of multiple glazing are calculated by considering the inter-reflections between the component layers (Jones, 1980; Mitalas and Stephenson, 1962). These calculations are performed for both direct and diffuse radiation. However, if the window incorporates a blind, the radiation reflected by or transmitted through it is assumed to be diffuse whatever the nature of the source. This is because the slat surfaces are assumed to be diffusing rather than specular reflectors (Parmelee and Vild, 1953).

The following equations are derived from Figure 5.A11.11 where all layers are symmetrical, i.e. both surfaces of the layer have the same reflection and the specularly of the radiation is not changed by the layer. If any of the layers are asymmetrical, the equations become more complicated since they have to include the reflection of both surfaces of the layer. If any of the layers is a diffusing slatted blind, then the direct radiation equations need to include the diffuse properties of the elements for radiation that has been reflected by the blind(s). Examples for some of these situations are given elsewhere (Jones, 1980).

### Double glazing

TAR coefficients for double glazing, denoted by prime ('), are as follows:

$$T' = (T_o T_i) / (1 - R_o R_i) \quad (\text{A11.44})$$

$$A'_o = A_o + [(T_o A_o R_i) / (1 - R_o R_i)] \quad (\text{A11.45})$$

$$A'_i = (T_o A_i) / (1 - R_o R_i) \quad (\text{A11.46})$$

$$R' = 1 - T' - A'_o - A'_i \quad (\text{A11.47})$$

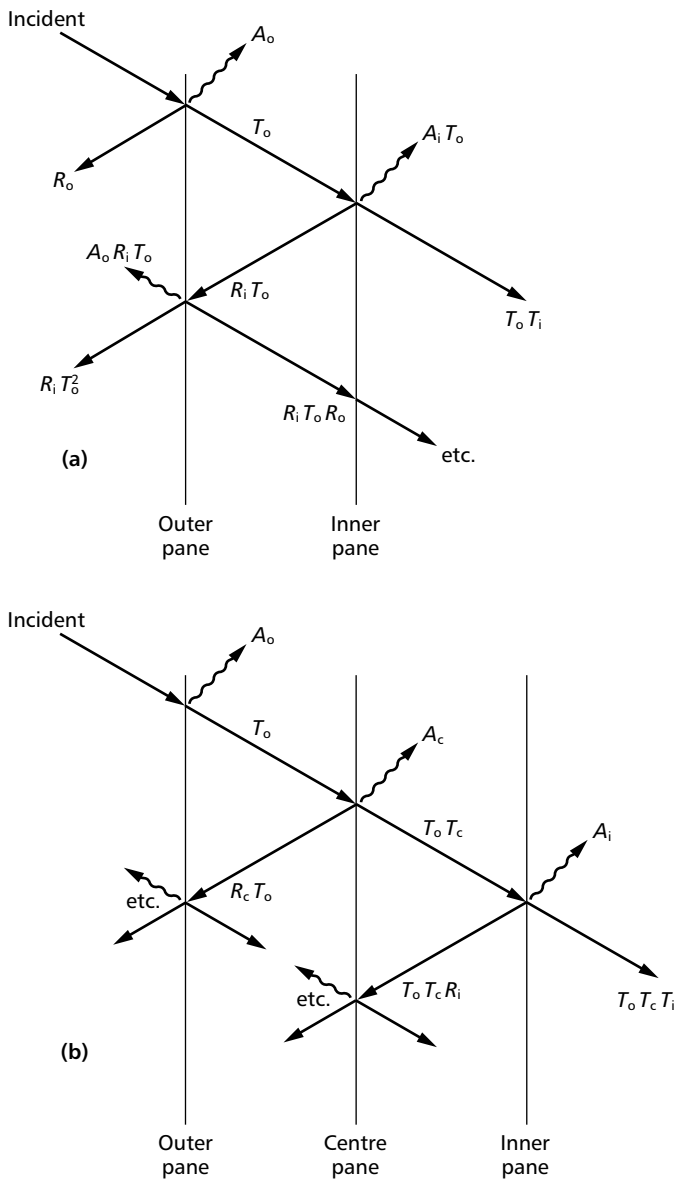


Figure 5.A11.11 Transmitted, absorbed and reflected radiation; (a) double glazing, (b) triple glazing

where subscript 'o' denotes the outer glazing element and subscript 'i' denotes the inner glazing element.

**Triple glazing**

TAR coefficients for triple glazing, denoted by double prime ("), are as follows:

$$T'' = \frac{T_o T_c T_i}{(1 - R_o R_c)(1 - R_c R_i) - T_c^2 R_o R_i} \quad (A11.48)$$

$$A_o'' = A_o + \frac{T_o A_o R_c}{1 - R_o R_c} + \frac{T_o T_c^2 A_o R_i}{(1 - R_o R_c)(1 - R_c R_i) - T_c^2 R_o R_i} \quad (A11.49)$$

$$A_c'' = \frac{T_o A_c (1 - R_c R_i + T_c R_i)}{(1 - R_o R_c)(1 - R_c R_i) - T_c^2 R_o R_i} \quad (A11.50)$$

$$A_i'' = \frac{T_o T_c A_i}{(1 - R_o R_c)(1 - R_c R_i) - T_c^2 R_o R_i} \quad (A11.51)$$

$$R'' = 1 - T'' - A_o'' - A_c'' - A_i'' \quad (A11.52)$$

where subscript 'o' denotes the outer glazing element, subscript 'c' denotes the central glazing element and subscript 'i' denotes the inner glazing element.

The heat gain to the environmental node due to conduction and radiation from the inner surface of the glazing is given by equation A11.7. If there is an internal blind, the additional heat gain to the air node is given by equation A11.9. In these equations, the transmittance factors ( $H$ ) depend on the values taken for the thermal resistances (i.e. the radiant and convective heat transfer coefficients) of the layers of the window. They are calculated by considering the thermal resistance network for the window. Figure

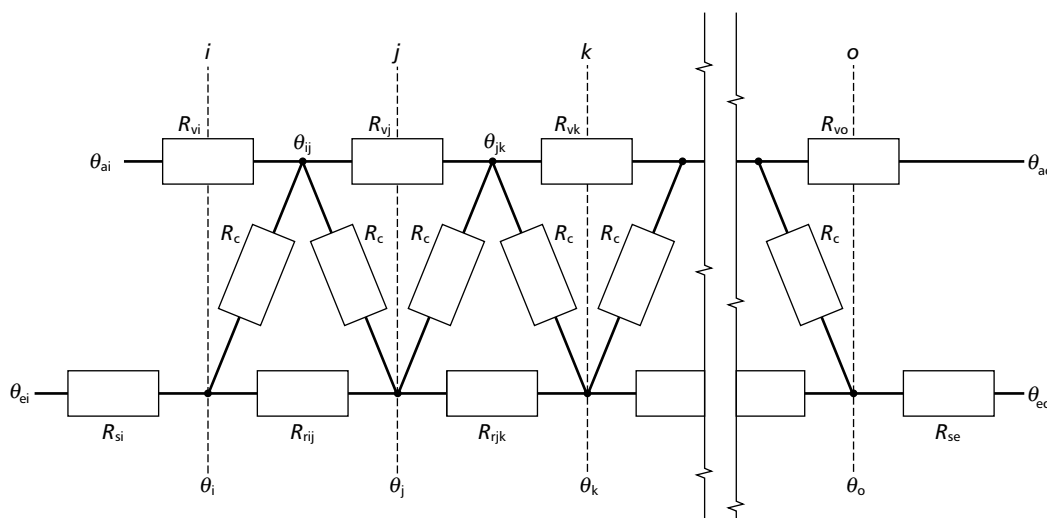


Figure 5.A11.12 General thermal resistance network for a multiple-layer window

5.A11.12 shows the general thermal resistance network for a multiple-layer window.

The properties of the glazing systems are calculated using the following standard thermal resistances and heat transfer coefficients:

- thermal resistance between inner surface of window and environmental point (i.e. inside thermal resistance):  $R_{si} = 0.12 \text{ m}^2 \cdot \text{K} \cdot \text{W}^{-1}$
- thermal resistance between outer surface of window and sol-air temperature (i.e. outside thermal resistance):  $R_{se} = 0.06 \text{ m}^2 \cdot \text{K} \cdot \text{W}^{-1}$
- convective resistance between a window layer and the air:  $R_c = 0.33 \text{ m}^2 \cdot \text{K} \cdot \text{W}^{-1}$  (for vertical window, corresponding to  $h_c = 3 \text{ W} \cdot \text{m}^2 \cdot \text{K}^{-1}$ )
- radiative resistance between two layers ( $j, k$ ) of window:
 
$$(R_{r,j,k}) = (\epsilon_j + \epsilon_k - \epsilon_j \epsilon_k) / (h_r \epsilon_j \epsilon_k) \quad (\text{A11.53})$$
 (if both layers have an emissivity of 0.84 and  $h_r = 5.7 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$ ,  $(R_{r,j,k}) = 0.24 \text{ m}^2 \cdot \text{K} \cdot \text{W}^{-1}$ )
- ventilation resistance across window layer between adjacent air spaces:  $R_v = 0 \text{ m}^2 \cdot \text{K} \cdot \text{W}^{-1}$  if the layer is a blind;  $R_v = \infty$  if the layer is glass.

**Example A11.1: Triple glazing without blinds**

Figure 5.A11.13 shows the network for triple glazing and Figure 5.A11.14 shows the simplified network resulting from evaluation of the parallel resistances.

The total resistance of the network is:

$$\begin{aligned} \Sigma R &= R_{si} + R_{ic} + R_{co} + R_{se} \\ &= 0.12 + 0.18 + 0.18 + 0.06 = 0.54 \text{ m}^2 \cdot \text{K} \cdot \text{W}^{-1} \end{aligned}$$

where  $R_{ic}$  is the thermal resistance between inner and central elements of the glazing ( $\text{m}^2 \cdot \text{K} \cdot \text{W}^{-1}$ ) and  $R_{co}$  is the thermal resistance between central and outer elements of the glazing ( $\text{m}^2 \cdot \text{K} \cdot \text{W}^{-1}$ ).

The transmittance factors for the inner, central and outer elements of the glazing can be shown to be:

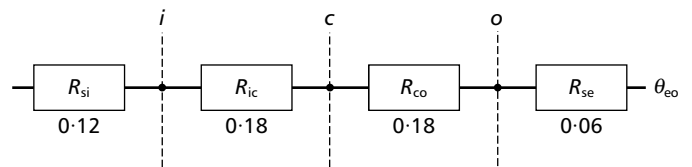
$$\begin{aligned} H_{ei} &= (R_{ic} + R_{co} + R_{se}) / \Sigma R \\ &= (0.18 + 0.18 + 0.06) / 0.54 = 0.78 \end{aligned}$$

$$\begin{aligned} H_{ec} &= (R_{co} + R_{se}) / \Sigma R \\ &= (0.18 + 0.06) / 0.54 = 0.44 \end{aligned}$$

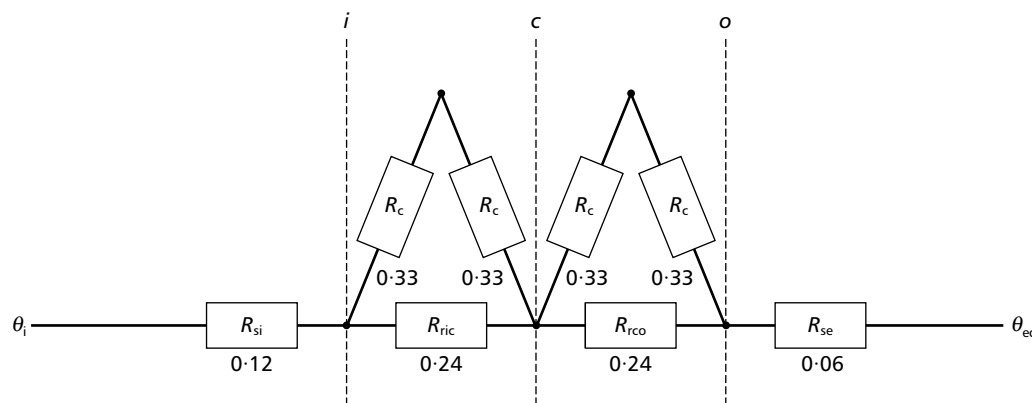
$$H_{eo} = R_{se} / \Sigma R = 0.06 / 0.54 = 0.11$$

From equation A11.7, the cyclic component of the convective and longwave radiant gain from the glazing to the environmental node is calculated as follows:

$$\tilde{H}_e A = H_{ei} \tilde{A}_i + H_{ec} \tilde{A}_c + H_{eo} \tilde{A}_o$$



**Figure 5.A11.14** Simplified thermal resistance network for triple glazing



**Figure 5.A11.13** Thermal resistance network for triple glazing

**Table 5.A11.3** Example A11.1: components of radiation

Time / h	Solar irradiance / $\text{W} \cdot \text{m}^{-2}$	Radiation absorbed/transmitted by glazing system / $\text{W} \cdot \text{m}^{-2}$				Gains to space / $\text{W} \cdot \text{m}^{-2}$		
		Radiation absorbed by inner, central and outer glazing elements			Directly transmitted radiation, $T$	Cyclic component of absorbed radiation, $(H_e A)$	Cyclic component of transmitted radiation for lightweight (L) and heavyweight (H) buildings	
		$A_i$	$A_c$	$A_o$			$\tilde{T}_L$	$\tilde{T}_H$
1200	442	35	58	89	133	37	—	—
1300	531	46	71	104	189	53	54	—
1400	572	52	77	108	223	60	99	34
1500	563	52	75	105	229	59	126	62
1600	504	47	67	94	205	50	131	79
Mean:	179	15	24	34	65	26	—	—

Table 5.A11.3 summarises the steps in the calculation of the solar gain to the space by means of an example. The calculation was carried out as follows.

For 12:00 h:

$$\begin{aligned}\tilde{H}_e A &= 0.78(35 - 15) + 0.44(58 - 24) \\ &+ 0.11(89 - 34) = 36.6\end{aligned}$$

The gain at other times is calculated similarly. The mean gain is calculated using the mean, rather than the cyclic, absorption values.

The cyclic component of the directly transmitted short-wave radiation is attenuated by the surface factor ( $F$ ), which is appropriate to the thermal weight of the building and corresponding time delay, see Table 5.A11.4.

For a lightweight building,  $F = 0.8$  and the time delay is 1 hour, i.e.:

$$\tilde{T}_L = 0.8 \times (T_{t+1} - \bar{T})$$

and for a heavyweight building,  $F = 0.5$  and the time delay is 2 hours, i.e.:

$$\tilde{T}_H = 0.5 \times (T_{t+2} - \bar{T})$$

where subscript 'L' denotes thermally lightweight building and subscript 'H' denotes thermally heavyweight building.

The mean solar gain factor is given by:

$$\bar{S}_e = \frac{\text{mean transmitted radiation plus mean absorbed radiation}}{\text{daily mean incident radiation}}$$

Hence:

$$\bar{S}_e = (65 + 26) / 179 = 0.51$$

The cyclic solar gain factors are calculated using the gains appropriate to a time one or two hours after the time of peak radiation, depending on the thermal weight of the structure, i.e.:

$$\tilde{S}_e = \frac{\text{total swing in gain to space}}{\text{swing in external gain}}$$

Peak solar irradiance occurs at 14:00 h; hence, for a thermally lightweight structure (i.e. 1-hour delay):

$$\tilde{S}_{eL} = (126 + 59) / (563 - 179) = 0.48$$

and for a thermally heavyweight structure (i.e. 2-hour delay):

$$\tilde{S}_{eH} = (79 + 50) / (504 - 179) = 0.4$$

*Example A11.2: Single glazing with internal absorbing blind*

Figure 5.A11.15 shows the network for single glazing with an internal blind and Figure 5.A11.16 shows the simplified network resulting from evaluation of the parallel resistances.

In this case, there are transmittance factors to both the air and environmental nodes, which are calculated as follows:

$$\begin{aligned}R_{ix} &= R_{rio} + [(R_c R_{se}) / (R_c + R_{se})] \\ &= 0.23 + [(0.33 \times 0.06) / (0.33 + 0.06)] = 0.28\end{aligned}$$

$$\begin{aligned}H_{ei} &= \frac{R_c R_{ix} / (R_c + R_{ix})}{R_{si} + [(R_c R_{ix}) / (R_c + R_{ix})]} \\ &= \frac{(0.33 \times 0.28) / (0.33 + 0.28)}{0.12 + [(0.33 \times 0.28) / (0.33 + 0.28)]} = 0.56\end{aligned}$$

$$\begin{aligned}H_{ai} &= \frac{R_{si} R_{ix} / (R_{si} + R_{ix})}{R_c + [(R_{si} R_{ix}) / (R_{si} + R_{ix})]} \\ &+ \left( \frac{R_{si} R_c / (R_{si} + R_c)}{R_{ix} + [(R_{si} R_c) / (R_{si} + R_c)]} \right) \left( \frac{R_{se}}{R_{se} + R_c} \right) \\ &= 0.24\end{aligned}$$

$$R_{ox} = R_{rio} + [(R_c R_{si}) / (R_c + R_{si})] = 0.32$$

$$\begin{aligned}H_{eo} &= \frac{R_c R_{se} / (R_c + R_{se})}{R_{ox} + [(R_c R_{se}) / (R_c + R_{se})]} \times \frac{R_c}{(R_c + R_{si})} \\ &= 0.10\end{aligned}$$

$$\begin{aligned}H_{ao} &= \frac{R_{se} R_{ox} / (R_{se} + R_{ox})}{R_c + [(R_{se} R_{ox}) / (R_{se} + R_{ox})]} \\ &+ \left( \frac{R_{se} R_c / (R_{se} + R_c)}{R_{ox} + [(R_{se} R_c) / (R_{se} + R_c)]} \right) \left( \frac{R_{si}}{R_c + R_{si}} \right) \\ &= 0.17\end{aligned}$$

**Table 5.A11.4** Thermal response

Thermal response	Typical features of construction	Response factor, $f_t$	Response to short-wave radiation		Time lead for admittance, $\omega$ / h
			Average surface factor, $F$	Time delay, $\phi$ / h	
Slow	Masonry external walls and Internal partitions, bare solid floors and ceilings	> 4	0.5	2	1
Fast	Lightweight external cladding, de-mountable partitions, suspended ceilings, solid floors with carpet or wood block finish or suspended floors	≤ 4	0.8	1	0

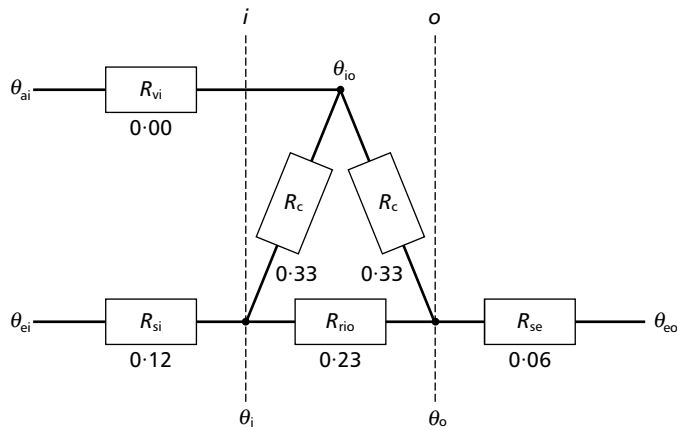


Figure 5.A11.15 Thermal resistance network for single glazing with internal blind

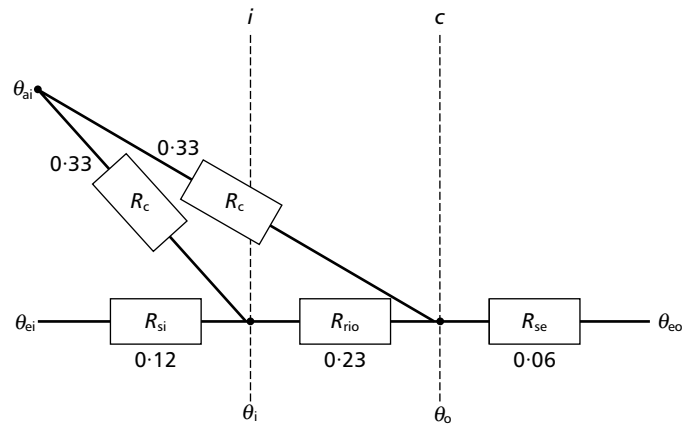


Figure 5.A11.16 Simplified thermal resistance network for single glazing with internal blind

Table 5.A11.5 Example A11.2: components of radiation

Time (h)	Solar irradiance / W·m <sup>-2</sup>	Radiation absorbed/transmitted by glazing system / W·m <sup>-2</sup>			Gains to space / W·m <sup>-2</sup>			
		Radiation absorbed by inner, and outer glazing elements		Directly transmitted radiation, T	Cyclic components of absorbed radiation		Cyclic components of transmitted radiation for lightweight (L) and heavyweight (H) buildings	
		A <sub>i</sub>	A <sub>o</sub>		H <sub>eA</sub>	H <sub>aA</sub>	T <sub>L</sub>	T <sub>H</sub>
1200	442	116	146	47	45	53	—	—
1300	531	151	178	52	68	67	22	—
1400	572	171	191	54	81	74	26	14
1500	563	173	186	51	81	73	28	17
1600	504	157	167	45	70	66	26	18
Mean:	179	51	58	19	34	22	—	—

Table 5.A11.5 summarises the steps in the calculation of the solar gain to the space.

The cyclic component of the convective and longwave radiant gain from the glazing to the environmental node is calculated as for triple glazing, see example A11.1, i.e.:

$$\tilde{H}_e A = H_{ei} \tilde{A}_i + H_{ec} \tilde{A}_c + H_{eo} \tilde{A}_o$$

The instantaneous component of the convective and longwave radiant gain from the glazing to the air node is calculated as follows:

$$H_a A = H_{ai} A_i + H_{ao} A_o$$

Hence, for 12:00 h:

$$H_a A = (0.24 \times 116) + (0.17 \times 146) = 52.66$$

The gain at other times is calculated similarly, see Table 5.A11.5. The mean gain is calculated using the mean, rather than the cyclic, absorption values.

The cyclic component of the directly transmitted shortwave radiation and attenuated by the surface factor (*F*) appropriate to the thermal weight of the building and delayed by a time corresponding to the thermal weight, see Table 5.A11.4.

For a lightweight building, *F* = 0.8 and the time delay is one hour, i.e.:

$$\tilde{T}_L = 0.8 \times (T_{t+1} - \bar{T})$$

For a heavyweight building, *F* = 0.5 and the time delay is two hours, i.e.:

$$\tilde{T}_H = 0.5 \times (T_{t+2} - \bar{T})$$

where subscript ‘L’ denotes thermally lightweight building and subscript ‘H’ denotes thermally heavyweight building.

As for example A11.1, the mean solar gain factor at the environmental node is given by:

$$\bar{S}_e = \frac{\text{mean transmitted radiation plus mean absorbed radiation}}{\text{daily mean incident radiation}}$$

Hence:

$$\bar{S}_e = (19 + 34) / 179 = 0.3$$

Again, as for example A11.1, the cyclic solar gain factors at the environmental node are calculated using the gains appropriate to a time depending on the thermal weight of the structure, i.e.:

$$\tilde{S}_e = \frac{\text{total swing in gain to space}}{\text{swing in external gain}}$$

Peak solar irradiance occurs at 14:00; hence, for a thermally lightweight structure (i.e. 1-hour delay):

$$\tilde{S}_{eL} = (28 + 81) / (563 - 179) = 0.28$$

and for a thermally heavyweight structure (i.e. 2-hour delay):

$$\tilde{S}_{eH} = (18 + 70) / (504 - 179) = 0.27$$

The mean solar gain factor at the air node is given by:

$$\bar{S}_a = \frac{\text{mean gain to air node}}{\text{mean incident radiation}}$$

Hence:

$$\bar{S}_a = 22 / 179 = 0.12$$

The cyclic solar gain factor at the air node is given by:

$$\tilde{S}_a = \frac{\text{total swing in gain to air node}}{\text{swing in external gain}}$$

There is no time delay associated with the air node, hence:

$$\tilde{S}_a = 74 / (572 - 179) = 0.19$$

### 5.A11.6 Shading coefficients

In addition to solar gain factors, Table 5.A11.1 also gives the shortwave and longwave shading coefficients ( $S_c$ ). These correspond to the direct and indirect transmission to the space for direct radiation at normal incidence (see 5.6.2) divided by ( $0.87 \times$  total incident radiation), where 0.87 is transmission coefficient of nominal 4 mm plain glass at normal incidence. However, it should be noted that the room gains, and hence the solar gain factors, depend on the direct and diffuse components of the incident radiation and the angle of incidence of the direct radiation.

The  $G$ -value does not distinguish between the two components of the transmission nor make reference to clear glass. Numerically, therefore, it is equal to the sum of the shortwave and longwave shading coefficients multiplied by 0.87.

## References for Appendix 5.A11

BSI (2006a) BS EN ISO 10077-1: 2006: *Thermal performance of windows, doors and shutters. Calculation of thermal transmittance. General* (London: British Standards Institution)

BSI (2006b) BS EN 13947: 2006: *Thermal performance of curtain walling. Calculation of thermal transmittance* (London: British Standards Institution)

BSI (2011) BS EN 673: 2011: *Glass in building. Determination of thermal transmittance (U-value). Calculation method* (London: British Standards Institution)

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Jones RHL (1980) 'Solar radiation through windows — theory and equations' *Building Serv. Eng. Res. Technol.* 1(2) 83–91

Parmelee GV and Vild DJ (1953) 'Design data for slat-type sun shades for use in load estimating' *ASHVE Trans.* 59 1403–1434

Nicol JF (1966) 'Radiation transmission characteristics of louver systems' *Building Science* 1 167–182

Mitalas GP and Stephenson DG (1962) *Absorption and transmission of thermal radiation by single and double glazed windows* Research Paper No. 173 (National Research Council of Canada, Division of Building Research)

Parmelee GV and Aubele WW (1952) 'The shading of sunlit glass' *ASHVE Trans.* 58 377–395

## Appendix 5.A12: Specification for reference (dynamic) model

In order to satisfy the requirements of the reference (dynamic) model, the features indicated below should be incorporated. This specification is not exhaustive but gives sufficient detail to provide a basis for assessing computer models.

### 5.A12.1 Analytical method

Calculations should be carried out for time increments not exceeding one hour using appropriate time sequences of climatic data, internal load patterns and required control set points. These may be hourly average values or, if the calculation requires a time increment of less than one hour, measured data corresponding to the time increment should be used or values may be interpolated from hourly data.

### 5.A12.2 Climatic data

Data for the following parameters are required at time increments not exceeding one hour:

- dry bulb temperature
- moisture content (or equivalent)
- solar irradiation, comprising direct, sky diffuse, ground reflected (taking account of site factors), sky temperature (or other parameter appropriate to the determination of longwave radiation from external surfaces)
- wind speed
- wind direction.

The effect on the convective heat transfer coefficient of wind speed and direction should be taken into account.

The solar component should include longwave radiation transfer to the sky and surroundings.

The conversion of solar irradiance data measured at a particular orientation and slope into values for other orientations and slopes should be achieved using the methods described in the annex to chapter 2 of this Guide: *Solar radiation, longwave radiation and daylight* (CIBSE, 2015).

Solar altitude and azimuth should be determined using the methods contained in the annex to chapter 2 of this Guide: *Solar radiation, longwave radiation and daylight* (CIBSE, 2015).

The conversion of measured climatic data into the form required by the calculation procedure should be achieved using the relationships given in chapter 1 of CIBSE Guide C: *Reference data* (2007).

### 5.A12.3 Properties of opaque fabric

The following properties should be represented:

- thermal resistance

- thermal capacitance
- surface emissivity (at boundaries and internal cavities)
- surface absorption coefficient for shortwave radiation
- convective and radiant heat transfer characteristics within cavities.

The dynamic response of opaque components may be determined using finite difference techniques or by response factors; other methods may be used provided that it may be demonstrated that they can achieve equal precision (CIBSE, 1998).

### 5.A12.4 Glazing

The following properties should be represented:

- thermal resistance
- solar absorption
- solar transmission
- surface emissivity
- convective and radiant heat transfer characteristics within internal cavities.

The performance of glazing systems should be based on the values of solar altitude and azimuth calculated at the solar time corresponding to the time for which the calculation is being performed. This may differ due to longitude and/or the effect of local adjustments for daylight saving.

The performance of glazing systems must take account of reflections between the elements comprising the system.

Separate calculations must be made for shaded and unshaded areas of glazed surfaces.

### 5.A12.5 Shading

Shading devices may consist of purpose built overhangs, side fins adjacent to or part of a window or moveable devices such as blinds, shutters or curtains.

The shading effect should be calculated for time increments not exceeding one hour using values of solar altitude and azimuth at the appropriate solar time. Where shading devices may be adjusted or controlled the effect of such features should be represented.

The model should take account of the effect of shading on glazing performance, as follows:

- in the case of purpose built shades, the determination of the amount and location of shade falling on the glazing; reflected radiation from the shades should also be considered
- for blinds and curtains, the absorbed and transmitted radiation to be calculated, if appro-

priate, as a function of slat angle; the interaction between glazing elements and blinds due to reflection of radiation from blinds must be represented.

Other obstacles to radiation such as shading by adjacent buildings and other site features should also be included, as should self-shading by the building under analysis.

### 5.A12.6 Internal longwave radiation

Longwave radiant heat transfer between surfaces and convective heat transfer between room air and room surfaces should be modelled using the fundamental heat balance described in Appendix 5.A2.

Longwave interchange between sources of internal heat gain and room surfaces must be modelled. The location of heat emitters should be taken into account.

### 5.A12.7 Internal shortwave radiation (direct solar gain)

The distribution of shortwave energy should be determined by calculation of the amount of direct and diffuse transmitted solar radiation incident upon each room surface. If a surface transmits shortwave radiation the quantity transmitted must be calculated using the same methods as for the transmission of solar radiation into the building. Reflections of shortwave radiation should be modelled.

The solar distribution must be calculated at the same frequency as that for the climatic data.

### 5.A12.8 Room air model

Convective heat gains may be assumed to enter directly into the air. The convective heat balance should include a representation of the thermal capacity of the room air. Under some circumstances it may be appropriate to increase the heat storage capacity of the air artificially to take

account of furnishings. However, there is little guidance available on when this is necessary.

The convective heat transfer coefficient at room surfaces should be calculated as a function of surface and air temperatures; suitable correlations are given by Alamdari (1980) and Hatton (1995). It is not considered practicable at present to include the influence of room air movement patterns.

### 5.A12.9 Infiltration and ventilation

The needs of design models and simulation models differ in that, for design purposes, it is usual to specify the value of infiltration whereas simulation techniques require this parameter to be calculated. Furthermore, ventilation to remove excess heat gain is an essential factor in the calculation of overheating risk. One way to determine ventilation rates is by means of a zonal airflow model. See chapter 4 of this Guide for guidance on the calculation of natural ventilation rates. The program supplier should provide details of the method used and be able to justify the assumptions made in the model.

## References for Appendix 5.A12

CIBSE (1998) *Building energy and environmental modelling* CIBSE Applications Manual AM11 (London: Chartered Institution of Building Services Engineers)

CIBSE (2000) 'Properties of humid air' ch. 1 in CIBSE Guide C: *Reference data* (London: Chartered Institution of Building Services Engineers)

CIBSE (2015) *Solar radiation, longwave radiation and daylight* Annex to CIBSE Guide A chapter 2 (London: Chartered Institution of Building Services Engineers)

Almadari F and Hammond GP (1980) 'Improved data correlations for buoyancy driven convection in rooms' *Building Serv. Eng. Res. Technol.* 4(3) 106–112

Hatton A and Awbi HB (1995) 'Convective heat transfer in rooms' *Proc. Building Simulation '95, August 1995, Wisconsin, USA*